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Abstract

Chaos theory, originally a branch of modern physics holding that there is pattern to the seeming randomness of physical events occurring in the universe, is emerging as one of the most-discussed new paradigms in contemporary American thought, both legal and otherwise. ... Thus, any price change can only be the result of new information, the production of which is itself assumed to be random. ... Despite this different testing methodology, semi-strong efficiency depends on the validity of the random walk model, which depends in turn on empirical conclusions concerning the absence of statistical dependence in security price data. ... The distinction between nonlinear and linear systems goes well beyond noise theory, however, because noise theory itself is constrained by the efficiency paradigm, whereas nonlinear dynamics and chaos theory break from that context and imply a fundamentally different understanding of public capital market behavior with a broader perspective on investor and market behavior. ... Although noise theory makes the important point that psychological or emotional trading is likely to be a factor, noise theory models explain biased price changes on the grounds that risk-averse arbitrageurs will not correct the effects of such trading. ... The Article examined the historical roots of the ECMH and the random walk model of public capital market behavior, which the ECMH explains. ... If the random walk model is not an accurate account of public capital market behavior, then the ECMH is largely meaningless because it answers the wrong question. ...

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From Random Walks to Chaotic Crashes: The Linear Genealogy of the Efficient Capital Market Hypothesis

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Introduction

Chaos theory, originally a branch of modern physics holding that there is pattern to the seeming randomness of physical events occuring in the universe, is emerging as one of the most-discussed new paradigms in contemporary American thought, both legaln1 and otherwise.n2 This Article argues that chaos theory and its lessons mandate opening a new chapter in a voluminous corporate and securities [*548] law debate revolving around the efficient capital market hypothesis (ECMH), which for nearly two decades has been the context for debating corporate and securities law and policy. <=4> n3 The debate has been defined by interpretations of the semi-strong form of the ECMH - the claim that security prices fully reflect all publicly available information. As such, the debate has assumed as true and built upon the weak form of the ECMH - the claim that security prices fully reflect all information consisting of past security prices. This Article analyzes the historical development of the ECMH, showing that the weak and semi-strong forms of the ECMH are based on linear methodology and thought that have been rendered obsolete by chaos models applying nonlinear techniques. <=5> n4 This obsolescence renders the ECMH false in all its forms.

The debate over the ECMH is fundamental because, despite interpretive questions concerning the semi-strong form, the ECMH is a major premise for a substantial body of corporate and securities law and scholarship. For example, the Securities and Exchange Commission has relied expressly on the ECMH in numerous contexts, including in promulgating its rules establishing the integrated-disclosure system and its rules authorizing shelf registration of securities. <=6> n5 In addition, the United States Supreme Court has recognized the ECMH as a basis for satisfying the reliance requirement in certain securities-fraud cases. <=7> n6 Moreover, since the late 1970s, a great deal of corporate and [*549] securities law scholarship has extolled the virtues of the ECMH and urged it as a basis to advocate many major policy prescriptions. <=8> n7 Indeed, before the public capital market crash of October 1987, only a few sobering pieces stood to remind the legal community that the ECMH is only a hypothesis, and a dubious one at that. <=9> n8 The crash inspired a long overdue reexamination of the ECMH in the legal literature. <=10> n9 Although important, that ongoing reexamination mistakenly [*550] accepts the weak form of the ECMH as a given and therefore permits the terms of the debate to continue to be set by interpretations of the semi-strong form.

One reason that the ECMH continues to dominate legal scholarship is the belief that the ECMH is based on "scientific evidence" and the argument that anyone who objects to it must rebut the proof with equally powerful "scientific evidence." <=11> n10 Although as a matter of the philosophy of science this is not the proper allocation of the burden of proof, this Article nevertheless invokes a scientific method that meets the burden of proof against the ECMH; it does so, however, in the spirit of introducing contemporary methodological shifts in financial economic theory that lawyers should feel obligated to confront sooner rather than later, without regard to the burden of proof. <=12> n11

This Article begins by exposing faults in the weak form of the ECMH. <=13> n12 Part I contains an historical account of the ECMH and shows that the weak form of the ECMH - commonly equated with the random walk model of security price behavior - originated as an explanation for the results of linear statistical models showing that security price behavior was random. <=14> n13 Economists developed those models in the 1960s using the first wave of high-speed computer technology. The entire ECMH depends on these tests. Part I also discusses noise theory, which raised some seemingly important but ultimately modest [*551] questions about the integrity of the ECMH by distinguishing fundamental efficiency from informational efficiency.

Part II demonstrates that the early techniques undergirding the ECMH are obsolete, having been superseded by modern statistical models using nonlinear mathematics and more sophisticated computer technology that enable analysts to model market behavior more accurately. The results of applying the new techniques contradict the earlier findings, instead showing that security price behavior is not random and thus that the ECMH is false.

Finally, Part III brings the history up to date, showing how chaos theory can account for market crashes, which neither the ECMH nor noise theory can do. It then demonstrates that the ECMH and noise theory - the two prominent capital market theories invoked in corporate and securities law discourse - represent an artificial continuum defined in terms of relative efficiency and relative rationality that are constrained by a linear frame of reference. Chaos theory breaks out of that antiquated paradigm to offer capital market rationales for such basic corporate and securities law doctrines as mandatory disclosure rules and mandatory fiduciary obligation, which neither the ECMH nor noise theory can do. The lessons of chaos theory also offer important insights into other debates, including capital market circuit breakers and relational investing. This Article concludes on a cautious note: It warns against excessive reliance on capital market theory - whether the ECMH, noise theory, or chaos theory - as a touchstone for policy formulation.

I. From Randomness to Efficiency

Economists developed the efficient capital market hypothesis in the mid-1960s to explain several empirical studies that some economists thought proved successive changes in security prices are random. Based on those studies, many economists thought that there is no pattern to the price history of a security and therefore that there can be no accurate prediction of future changes in security prices based on prior prices. This lack of pattern was the basis of the random walk model of security price behavior. The ECMH explains that model by hypothesizing that price changes occur as the result of changes in information concerning the security in question.

A. The History of the Random Walk Model

1. Obscurity

The early history of the random walk model is itself somewhat haphazard. The model dates to 1900, when it was elaborated in a then- [*552] obscure, now famous, doctoral dissertation by the French mathematician Louis Bachelier. <=15> n14 That dissertation investigated linear correlation in prices of options and futures traded on the French Bourse and concluded that such price changes behaved according to a random walk model. <=16> n15

Bachelier's dissertation was not widely noticed when published, perhaps because "his accompanying development of the mathematical theory of random processes anticipated by 5 years Einstein's famous work on the random motion of colliding gas molecules." <=17> n16 Einstein's work on this subject consisted of "discovering" the equation that describes the phenomenon of random molecular motion, known as Brownian motion (after the Scottish botanist Robert Brown, who first observed it), which was precisely the equation Bachelier developed to describe price behavior in financial markets. <=18> n17

Although the mathematical properties Bachelier proved were of direct and immediate interest to physicists and mathematicians (including Einstein and his intellectual progeny), economists paid little attention to the subject until the middle of the century. <=19> n18 Indeed, virtually no studies prior to the early 1950s made any reference to Bachelier's work or to the theory of random processes in financial markets. <=20> n19

Maurice Kendall is frequently credited with bringing the random walk model to the attention of economists in the early 1950s. <=21> n20 Bachelier's work itself, however, was not "discovered" by economists until Paul Samuelson and his colleagues at MIT stumbled across it in the [*553] mid-1950s, <=22> n21 and the "main modern interest" in the random walk model did not begin until 1959, when it became a very popular area of research. <=23> n22

2. Linear Testing

In the early 1960s, many capital market theorists began exploring random processes in public capital markets. <=24> n23 In their empirical research, these theorists tested the hypothesis that successive price changes were random. <=25> n24 They used only two kinds of statistical tests: linear autocorrelation tests and linear run analysis. <=26> n25 In addition, some researchers investigated whether trading rules could be developed to achieve above-normal returns. <=27> n26 [*554]

a. Autocorrelation Tests

Autocorrelation, sometimes called serial correlation analysis, is used to determine whether specified data sequences move together to any degree. <=28> n27 In the case of security prices, price changes of a given security are recorded over a specified time period - often, a number of days - and a subsequent time period of the same length. These sequences (called time-series data) are then compared and examined to determine whether they move together to any degree - that is, whether there is any "serial correlation." <=29> n28 A "correlation coefficient" is mathematically derived to express the degree to which the data are linearly related. <=30> n29 In effect, the time series of data is tested for correlation by fitting a straight line to the data and then calculating the correlation coefficient. <=31> n30 A correlation coefficient equal to zero is evidence that the data in the series has the property of statistical independence; correlation coefficients that are close to zero (but not equal to zero) indicate that the data were uncorrelated. <=32> n31 A time series of data is random if it is either independent or uncorrelated. <=33> n32 [*555]

The autocorrelation analyses of the 1960s all resulted in correlation coefficients that did not differ significantly from zero. <=34> n33 This meant that various series of actual securities-market data were indistinguishable from various series of numbers generated by a random-number table, roulette wheel, or other device of chance. <=35> n34 These findings had an important practical implication: traders could not systematically make gains from speculative trading because a statistical lack of correlation implies that the best estimate of the future price of a security is its present price. In other words, "if prices follow a random walk, the price change from t-1 to t will not affect the probability that a particular price change will occur at period t+1." <=36> n35 Thus, past prices cannot help to predict future prices.

b. Run Analysis

One long-known weakness of the statistical technique of autocorrelation is that the results can be skewed by a small number of extraordinary data in the time series. <=37> n36 An alternative test that avoids this weakness is an analysis of runs in the data - an investigation of whether there is any persistence to the direction of successive changes. <=38> n37 Thus, instead of testing the correlation of numerical [*556] changes in the data in the series, the relationship of the direction of those changes is investigated. If price changes follow the random walk model, the number of sequences and reversals in time series data of security prices would be roughly equal. If the same direction persisted for a significantly longer period, however, then the random walk model would be contradicted.

Of the numerous run studies conducted in the early 1960s, Professor Fama's is regarded as the most careful. <=39> n38 Professor Fama found that the direction of price changes tended to persist but nevertheless concluded that no trading rule or strategy could be derived that outperformed the market consistently. <=40> n39 Accordingly, almost everyone involved in the debate in the late 1960s agreed that the observed departures from randomness were negligible and believed that this constituted strong support for the random walk model. <=41> n40

c. Trading Rule Tests

Despite the widespread agreement, some participants in the debate remained skeptical. Indeed, prescient commentators of the era occasionally expressed the fear "that the interrelationships of price changes are so complicated that standard statistical tools, such as serial correlations, will not find them." <=42> n41 This fear led to efforts to dispute the model by designing trading rules that could achieve above-normal returns by uncovering and exploiting these greater complexities. Among the most primitive, though most illustrative, trading rules was Professor Alexander's "filter technique." <=43> n42 This is a strategy designed to discern and exploit assumed trends in security prices that, [*557] in Alexander's piquant phrase, may be "masked by the jiggling of the market." <=44> n43 Consider the following example. <=45> n44

A simplified "five percent filter rule" with respect to a specified security would provide as follows: <=46> n45

(1) At such time as the price shall have increased by five percent above its current price (on its way to some higher peak), buy; then

(2) hold; then

(3) at such time as the price shall have decreased by five percent from the peak (on its way to some lower trough), (a) sell (realizing a gain on the sale), <=47> n46 and (b) go short; <=48> n47 then

(4) at such time as the price shall have increased by five percent from the trough, (a) buy, and (b) cover the short position <=49> n48 (realizing a gain on the covering). <=50> n49

The success of this technique would imply that price changes follow the peak-trough pattern that the filter is detecting and exploiting. Those changes would by definition not be random; therefore, the random walk model would be contradicted. Alexander's initial results indicated that such a technique could produce above-normal returns. <=51> n50 Subsequent refinements of Alexander's work by himself <=52> n51 and other scholars, including Eugene Fama, however, demonstrated that relaxing or changing certain assumptions eliminated the abnormal returns. <=53> n52 [*558]

Alexander's filter technique epitomizes the chartist or technical approach to security analysis and trading, under which a study of past prices (or other data) is used as a basis for predicting future prices. Indeed, Alexander's filter technique is a conceptual cousin of limit orders and similar techniques prevalent in securities trading today. <=54> n53 Such techniques are staples of Wall Street futurology: They commonly are used by traders and recommended by investment advisers and brokers, notwithstanding the view of many students of the random walk model (and the ECMH) that, based on the foregoing, they are nonsense. <=55> n54 On the other hand, the autocorrelation and run studies admittedly rely solely on linear models and do not investigate the presence of nonlinear dependence. <=56> n55 Indeed, the foregoing trading-rule test operates in chronological time (or real time) - a linear concept - without considering the possibility that market or economic time may be nonlinear. <=57> n56

B. Development of the ECMH

Many have suggested that the ECMH developed in a peculiar manner uncommon in scientific development: <=58> n57 The proof of the hypothesis came first, beginning with Bachelier in 1900 and proceeding through the wealth of studies reporting randomness in the early 1960s. Only then was a theory proposed to explain the randomness, beginning with the first explication of the ECMH by Paul Samuelson in 1965. <=59> n58 Economists welcomed this proof: the conditions necessary [*559] to produce it seemed tantalizingly near to those necessary to sustain a perfect market.

For economists seeking empirical support concerning market conditions approximating the theoretical perfect market, the proof supporting the random walk model was very rich indeed. The perfect market is a heuristic invented by stipulating the following assumptions concerning a market: there are a large number of participants such that the actions of any individual participant cannot materially affect the market; participants are fully informed, have equal access to the market, and act rationally; the commodity is homogeneous; and there are no transaction costs. <=60> n59 Under these assumptions, the perfect market model would predict precisely what the random walk model was implying: that prices of goods (securities) in the public capital markets should adjust instantaneously and accurately to new information concerning them. <=61> n60 That prediction was embodied in the ECMH as first propounded: in its broadest terms, the ECMH held that the prices of securities traded in public capital markets fully reflect all information concerning those securities.

1. Three Forms of Efficiency

Under this broad statement, the ECMH explains more than the random walk model required: the random walk model holds simply that successive price changes are independent or uncorrelated, whereas [*560] the ECMH explains that holding by saying that public security prices fully reflect all information - not just price histories - about a security. As a result, virtually since the emergence of the ECMH as an explanation of the random walk model, the ECMH has been divided into three forms, defined in terms of specified categories of information. <=62> n61 The three forms were first proposed to classify empirical tests of price behavior given specified kinds of information. <=63> n62 The weak form tested the random walk model itself, using autocorrelation tests and run analysis to investigate whether past prices indicate anything about future prices. Semi-strong form testing investigated whether publicly available information other than prices was reflected in prevailing prices, and strong-form testing investigated whether private information was reflected in prevailing prices.

As the wealth of tests and discussion proceeded in the 1970s, the three forms of the ECMH came to be used to refer to the conclusions those tests suggested. <=64> n63 Thus, the forms of the ECMH are now specified as follows: the weak form holds that current security prices fully reflect all information consisting of past security prices; the semi-strong form holds that current security prices fully reflect all information that is currently publicly available; and the strong form holds that current security prices fully reflect all currently existing information, whether publicly available or not. <=65> n64 Finally, each of these three forms may be respecified in terms of their implications: a market is efficient with respect to an information set (defined, for example, as price histories, publicly available information, or private information) when it is not possible to generate above-normal returns by trading on the basis of that information set. <=66> n65

There is thus a direct and logical link between the random walk model and weak-form efficiency <=67> n66 and a more attenuated and contingent link between the random walk model and stronger forms of the [*561] ECMH. Recall that the random walk model held that price changes are independent of or uncorrelated with prior price changes. <=68> n67 That means that technical analysis of past price changes - sometimes called chartist analysis - cannot aid prediction of future price changes in any systematic way. Weak-form efficiency explains this independence and its implications for prediction by hypothesizing that the current price impounds all information contained in prior prices. Thus, any price change can only be the result of new information, the production of which is itself assumed to be random. <=69> n68 This process of information absorption continues and thus explains the absence of substantial linear dependence in successive price changes discovered in the autocorrelation and run tests described above. <=70> n69 It also leads to the stronger forms of the hypothesis.

The semi-strong form of the ECMH posits not only that current security prices reflect all information consisting of prior security prices but also that they reflect all publicly available information about the security in question. Testing this more ambitious claim requires a focus not on correlation analysis of price changes but on the relative swiftness with which prices change given new information. <=71> n70 Despite this different testing methodology, semi-strong efficiency depends on the validity of the random walk model, which depends in turn on empirical conclusions concerning the absence of statistical dependence in security price data. <=72> n71 In other words, if future price changes are [*562] dependent on prior price changes, then any price change the semi-strong form tests cannot be attributable solely to new information the test is evaluating. Thus, weak-form and semi-strong-form efficiency each depend on the proof provided by linear testing models.

The strong form of the ECMH extends much further than the random walk model suggested. Indeed, the strong form is a theological proposition, holding that public capital markets are infinitely wise: even non-public information is reflected in public security prices. Numerous studies have investigated whether persons possessing non-public information could, by using that information as a basis for trading decisions, achieve abnormal market returns. The theory is that, if market prices reflect less than all information, then those with the excess (private) information should be able to outperform the rest of the market. <=73> n72 The insider trading scandals of the 1980s <=74> n73 are among the many proofs that the strong form of the ECMH is invalid. <=75> n74

Because the strong form of the ECMH is widely discredited, debate concerning the ECMH centers on the semi-strong and weak forms. Debate over the weak form generally is defined by analysis of the random walk model itself, usually in terms of linear empirical models used to test the relationship between successive price changes, <=76> n75 [*563] although more recent attention is being paid to nonlinear testing and nonlinear models. <=77> n76 Debate over the semi-strong form traditionally has been defined in terms of anomaly studies - empirical models demonstrating that above-normal returns can be achieved by using specified information sets that are publicly available. <=78> n77 More recently, debate over the semi-strong form of the ECMH also has incorporated a revival of an old debate over the distinction between informational and fundamental efficiency, which is at the core of noise theory.

2. Noise Theory and Efficiency

In a famous paper, Nobel Laureate James Tobin suggested that even if a public capital market is efficient in the sense of swiftly incorporating public information into security prices (i.e. the semi-strong form of the ECMH), that does not necessarily mean that security prices in that market reflect fundamental values (i.e. the present value of expected future flows to securityholders). <=79> n78 The conceptual distinction Professor Tobin makes between informational and fundamental [*564] efficiency dates to Keynes's well-known beauty-contest metaphor in which each judge picks the candidate that he or she thinks others will pick - a non-fundamental method - rather than the candidate that he or she thinks should win on the merits - a fundamental method. <=80> n79 If Professor Tobin is right, then the semi-strong form of the ECMH itself would have to be subdivided and evaluated separately with respect to strict informational efficiency and a more refined notion of fundamental efficiency. <=81> n80

In this context, informational efficiency describes a market in which all public information about a security is reflected in the price of that security, without regard to the quality of that information. Thus, information that concerns the fundamental value of a security is reflected, but so is information wholly unrelated to that fundamental value, such as who won the Super Bowl. <=82> n81 Fundamental or allocative efficiency, then, is the more narrow proposition that security prices are accurate indicators of intrinsic value because they reflect strictly information concerning fundamental asset values. <=83> n82

The technical issue thus becomes whether the information-processing function of public capital markets is capable of distinguishing [*565] among kinds of information such that only information about fundamental value is impounded and reflected. <=84> n83 In human terms, the issue becomes the pervasive question of whether humans behave rationally. <=85> n84 The possibility that human beings behave irrationally has been resisted by economists for centuries, and is "assumed out" of the ECMH model. <=86> n85 The informational/fundamental distinction, however, is so potent - both intuitively and empirically - that it had to be confronted. The result was the face-saving shelter of euphemism: Fischer Black, borrowing a term from the statistics literature, renamed irrational behavior noise, thus enabling self-respecting economists to discuss the issue and to try to model it. <=87> n86

Noise theory is supported by substantial empirical evidence and a well-developed intellectual foundation. <=88> n87 Noise-theory models hold that the public capital markets are infected by a substantial volume of trading based on information unrelated to fundamental asset values (noise trading). <=89> n88 These models attempt to explain both why noise trading occurs and why its effects persist. <=90> n89 The most common noise-theory model holds, for example, that noise trading is conducted by [*566] ill-informed investors and that it keeps prices from reflecting fundamental values accurately because even sophisticated arbitrageurs will not fully arbitrage its influence away. <=91> n90 This is because arbitrageurs are risk averse, and they cannot be sure that the misperceptions of the noise traders will not change adversely at any time. <=92> n91

At first, the informational/fundamental distinction and the development of noise theory seem to subvert the ECMH. In fact, however, these concepts generally are treated as studies in behavioral economics that continue to be constrained by a linear frame of reference within the ECMH framework. <=93> n92 There is a more profound implication not generally recognized and certainly not taken to its logical conclusion: the fundamental/informational distinction and noise theory reveal that public capital markets are nonlinear systems. <=94> n93 The next Part takes that nonlinear reality to its logical and revolutionary conclusion; first, however, two important implications of the ECMH for asset pricing theory warrant discussion.

C. Role of Asset Pricing Theory

The ECMH holds that specified information sets are fully reflected in the price of public securities. The ECMH, however, does not provide any basis for determining what it means for any such information to be fully reflected in security prices. Therefore, market efficiency analysis requires a separate model, called a pricing model, to define what fully reflected means in this context. <=95> n94 This requirement has an important consequence: tests of the ECMH are simultaneously tests of a pricing hypothesis, and the two hypotheses are tested jointly. <=96> n95 [*567] Accordingly, although a full discussion of asset pricing theory is beyond the scope of this Article, a brief account of two highlights is necessary: modern portfolio theory, which provides the foundation for asset pricing theory, and the capital asset pricing model, which is the general paradigm of asset pricing theory.

1. Modern Portfolio Theory

While the random walk model was being developed in the 1950s, Harry Markowitz was developing "modern portfolio theory" (MPT). <=97> n96 MPT proposes that all investments are reducible to two elements, risk and return, and assumes that investors are risk averse in the sense that they will sacrifice returns to avoid risk and demand greater returns to assume risk. <=98> n97 MPT holds that such investors will best address their risk aversion by investing in a portfolio of investments in which they receive the greatest expected return for any given level of risk.

Under MPT, expected return on an investment is simply the weighted arithmetic average of all possible returns on it, and the risk of an investment is the dispersion of possible returns on that investment around the expected return. <=99> n98 Under MPT, expected return on a portfolio of investments is simply the weighted sum of the expected returns on the individual investments; the risk, however, of a portfolio of investments is not necessarily the weighted sum of the risks (or dispersion in the returns) of the individual investments. <=100> n99 This is the central insight of MPT: because variations in returns on individual investments may reduce the dispersion of returns on a portfolio of investments, portfolio risk is primarily a function of the degree of variance of individual investments compared to the portfolio as a whole. <=101> n100 This function is called the covariance and means that portfolio risk is minimized through portfolio diversification. <=102> n101

MPT's understanding of risk has another important implication. With respect to any security, two elements of risk can be distinguished: [*568] systematic and unsystematic risk. Systematic risk arises from the tendency of a security to vary as the market in which it is traded varies. <=103> n102 Unsystematic risk arises from the peculiarities of the particular security being investigated. <=104> n103 Because under MPT's diversification directive unsystematic risks can be diversified away to zero, market returns on a security in a competitive market will not include any compensation for such risk. <=105> n104 Thus, market returns will be a function solely of the systematic risk, or the extent to which a particular security varies as the market (or system) of which it is a part varies. Measuring such risk and return is one goal of capital asset pricing models.

2. The Capital Asset Pricing Model

As MPT and the random walk model were maturing in the late 1960s, capital asset pricing theory was in its infancy. <=106> n105 The capital asset pricing model (CAPM) that is most widely known today derives from MPT. <=107> n106 Like MPT, CAPM assumes investors are risk averse in the sense just described. <=108> n107 In addition, CAPM assumes investors have rational expectations concerning expected returns. Under this assumption, CAPM holds that the expected return on an investment is equal to the risk-free rate of return <=109> n108 plus compensation for the systematic risk of the investment in the sense just described. That systematic risk is measured by the degree of variability of the individual investment versus the market as a whole. This measure is expressed as a number, called b. Thus, formally, CAPM is specified as follows:

(r<s> - r<f>) = b(r<m> - r<f>)

[*569] where r<s> is the expected return on a security; r<f> is the risk-free interest rate (and together (r<s> - r<f>) is the expected premium); b is the covariance of the security with respect to the market as a whole; and r<m> is the expected return on the market (and (r<m> - r<f>) together is the expected market risk premium). <=110> n109 Under CAPM, securities with higher bs are more risky than securities with lower bs because they tend to swing more widely than the market; their returns exhibit greater dispersion versus market returns.

Two important observations about MPT and CAPM are warranted. First, in evaluating the ECMH, the need for a pricing model creates the joint-hypothesis problem: one can never be certain in testing a model whether rejections of it are required because of market inefficiency or because of an inadequately specified asset pricing model. <=111> n110 Indeed, many of the anomalies discovered over the past twenty years in the ECMH <=112> n111 are attributed to deficiencies in the asset pricing model rather than to the presence of market inefficiency. <=113> n112 These deficiencies are most often associated with imprecision in defining risk or, equivalently, in specifying b.

The joint-hypothesis problem has an important implication for those who hold deep suspicions about market efficiency and wish to disprove it: to disprove the ECMH requires proof that does not invoke any asset pricing model - indeed, as indicated above, invoking an asset pricing model is the sure route to the anomaly heap. By hypothesis, however - under any existing asset pricing model, including CAPM - any linear or nonlinear dependence in security price behavior is inconsistent with the ECMH itself. Thus, a discovery of linear or nonlinear dependence in successive security prices would indicate that the ECMH was an incomplete explanation and would not admit the alternative explanation of a "misspecified" asset pricing model. <=114> n113

Second, CAPM specifies that expected returns from an investment are linearly related to expected returns on the portfolio of which that investment is a part. <=115> n114 The linear relationship is specified by b and that specification in turn is dictated by CAPM's rational-expectations assumption. <=116> n115 If human behavior is itself inconsistent with the rational-expectations assumption, then there is no reason to believe in such a linear relationship. <=117> n116 This would be another way of saying that [*570] the market system is nonlinear rather than linear. In that case, b will not be an accurate measure of risk.

It is possible that this weakness in b can be addressed by some techniques being developed in nonlinear dynamics that will be discussed in Part II. <=118> n117 Before beginning Part II, however, there is another notable dimension of the rational-expectations assumption used in the CAPM. That assumption is that investors have homogeneous return expectations; this in turn requires that investors evaluate and understand information in identical ways. <=119> n118 Furthermore, it would require all investors to evaluate investment opportunities based on identical time horizons. <=120> n119 The patent dubiousness of these requirements recently has become an important part of the literature criticizing CAPM. <=121> n120 That literature demonstrates that demand curves for particular securities are in fact downward-sloping: different investors have different appetites for particular securities at different prices. <=122> n121 Thus, markets do not depict the right price of a security because there is no such thing. This is because human beings - even rational ones - are not homogeneous automatons. <=123> n122

Recall in this connection that the random walk model got that name because public capital markets seemed to obey the principles of Brownian motion, which specify that molecules in motion behave randomly. <=124> n123 Although molecules lack sentience, prices are strictly creatures of the ultimate sentience, human behavior. Thus, intuition alone suggests that the price-molecule parallel should not hold. More powerfully, current thought in physics concerning nonlinear dynamics and chaos theory extends well beyond Brownian motion and suggests further reasons to doubt and reconsider the validity of the analogy. [*571]

II. From Efficiency to Chaos

The random walk model of security price behavior was an important innovation, developed and confirmed using the first wave of high-speed computer technology in the 1960s and relying on proof that successive price changes exhibit little or no linear dependence. The efficient capital market hypothesis was equally important as a way to explain this lack of dependence, or randomness, by hypothesizing that such price changes are a result of the availability of new information and that such information is rapidly impounded into the new price. This hypothesis was then divided into three strengths, defined in terms of categories of information that the market was processing. Although few informed students of the ECMH ever invested great confidence in the strong form, the semi-strong and weak forms have held sway over academic discourse in financial economics and law for nearly two decades. <=125> n124 Because those hypotheses are based exclusively on linear analysis and thought, <=126> n125 however, their descriptive power and normative implications have become questionable.

A. Nonlinear Dynamics

Noise theory shows that the information-processing properties of public capital markets are so bluntly powerful that fundamental information about underlying asset values is crowded out by extraneous information or noise. The presence of noise itself reveals the inadequacy of the linear testing models that led to the random walk model and the ECMH. The phenomena that the fundamental/informational distinction and noise theory explain constitute a feedback system in which individuals together overreact to information or withhold action in the face of information. Such feedback processes are hallmarks of a nonlinear system because they indicate a non-proportional relationship between a cause and its effect (e.g., between news and price changes). As noted above, however, this insight of noise theory has not been recognized for the full power of its implications, which is that the distinction between linear and nonlinear is fundamental to an understanding of public capital market behavior and the manner in which the law should treat the market and its participants.

1. Linear and Nonlinear Distinguished

Linearity means proportionality: a change in one variable produces a proportionate change in another specified variable. What makes the [*572] CAPM linear, for example, is its assertion that in a competitive market, the expected risk premium varies in direct proportion to b. <=127> n126 There are two senses in which the ECMH is linear. First, the statistical models underlying the weak form are simple linear regression analyses; correlation coefficients are statements about how variables are related on a straight line basis over time. In other words, the time series of data is tested for correlation by fitting a straight line to the data and then calculating the correlation coefficient. <=128> n127 Second, the insight underlying the semi-strong form of the ECMH is linear because it defines a proportional relationship between information changes and price changes. In particular, the semi-strong form posits that information is swiftly incorporated into prices without bias. In other words, there is a proportional relationship between information changes about underlying (real) asset values and resulting price changes in the financial asset representing those real assets.

In contrast, nonlinearity means the absence of proportionality - changes in one variable will produce a change in another variable but exponentially rather than proportionally. <=129> n128 To take a prosaic example, the one-ounce straw that breaks the one-ton camel's back is nonlinear because the cause is utterly disproportionate to the effect. In capital markets, plummeting stock prices and crashing markets have been attributed to an incremental bit of information piled on top of cumulated bits of information. <=130> n129 That the market may react slowly or may overreact to bits of new information is of course what noise theory teaches and explains. <=131> n130 The distinction between nonlinear and linear systems goes well beyond noise theory, however, because noise theory itself is constrained by the efficiency paradigm, whereas nonlinear dynamics and chaos theory break from that context and imply a fundamentally different understanding of public capital market behavior with a broader perspective on investor and market behavior.

2. Empirical Methodology and Studies

Because there is no a priori reason to believe that public capital markets are linear systems rather than nonlinear systems, one of the first questions that must be considered in understanding such markets is whether they follow linear or nonlinear processes. More sophisticated techniques than were available when the random walk model was first [*573] developed are now used to investigate precisely that question. <=132> n131 Researchers start with the observation that "the consensus of published empirical research is that a geometric random walk describes prices fairly well, although some anomalies have been reported." <=133> n132 Then they dig deeper. <=134> n133

a. Rescaled Range Analysis

Rescaled range analysis (R/S Analysis) investigates whether observations of a variable taken over time follow a random process. <=135> n134 It was first developed for study in natural sciences. For example, consider a hydrologist developing reservoir-discharge policies to maintain reservoir water levels in the light of rainfall patterns. The hydrologist records reservoir water levels each day at noon and calculates the range of his observations. <=136> n135 If the water level follows a random process, then the range would increase with the square root of the number of observations. <=137> n136 In other words, the range would be a [*574] function of some constant multiplied by the number of observations raised to the power of .50. <=138> n137 The exponent may take on other values, however, and it is those values that would interest the hydrologist in developing discharge policies. That exponent is called the Hurst exponent or H, after H.E. Hurst, the hydrologist who developed it. <=139> n138

If H equals .50, then the system being measured behaves according to a random walk model. <=140> n139 If H is less than .50, the system is mean reverting: if the system has moved up for a number of observations, it is more likely to move down over the next number of observations, and vice versa. <=141> n140 Conversely, if H is greater than .50, the system is correlative or persistent: if the system has moved up for a number of observations it is more likely to continue to move up over the next number of observations, and vice versa. <=142> n141 H is thus an indirect measure of correlation. That correlation can be measured directly by the following equation:

C = 2[su'(2H - 1)'] - 1

where

C = correlation measure; H = Hurst exponent. <=143> n142

Thus, for the following values of H, C would have the specified values, as the foregoing explains:

H = 0.50, C = 0.00; H = 1.00, C = 1.00; H = 0.00, C = -0.50. <=144> n143

H therefore also measures the likelihood that a datum in a series will behave in a particular way. For example, if H is .50 (i.e., the system is random), then the probability that a positive move will follow a positive move is one half. But if H is .60 (i.e., the system is persistent), [*575] then the probability that a positive move will follow a positive move is sixty percent. <=145> n144

H may change over time. For example, H may be in the .70s over some period and then drop to near .50 and subsequently increase again. The number of observations (or time periods) over which H is sustained at other than .50 (before returning to near .50) is a measure of the average cycle of the system. <=146> n145 In the case when H exceeds .50 for a sustained period, the length T of that period is a measure of the system's memory cycle - the extent to which past events influence present and future events. In the context of investment analysis, it measures the period over which an investor can use information to his or her advantage. <=147> n146

Edgar Peters has applied R/S Analysis to numerous public capital markets. <=148> n147 For example, he applied R/S Analysis to the Standard and Poor's 500 Index (the S&P Index) for monthly data over a thirty-eight year period from January 1950 through July 1988. <=149> n148 Peters found that H = .78 for average periods of approximately four years, indicating a strong persistent element in the S&P Index rather than a random process. <=150> n149 Beyond average periods of four years, however, H was not significantly different from .50 (it was .52 /-.02). <=151> n150 Thus, Peters concludes that the system begins to lose memory of events after four years. <=152> n151 In other words, the S&P Index is not random and events today continue to affect price changes measurably for up to an average of four years.

As with all statistical measures, there are potential weaknesses to using H as a measure of nonlinear dependence in time-series data. How confident can we be that it is a reliable measure of randomness versus dependence at all? For example, H values different from .50 may say nothing about randomness or dependence but rather may indicate [*576] only that particular data scale to a different value than H = .50. Alternatively, H values may measure only the nature of the data distribution (e.g., whether they follow a normal distribution or have high peaks and fat tails). <=153> n152

To address these concerns, Peters scrambled the original S&P Index time-series data in his study and conducted a separate R/S Analysis on the scrambled data. If H were measuring only scaling or distributions, then the value of H derived from the original data would not differ significantly from the value of H derived from the scrambled data. <=154> n153 In Peters's scrambled R/S Analysis, H equaled .51 over the entire time period, meaning that "scrambling destroyed the long memory structure of the original series and turned it into an independent series." <=155> n154 This reinforced Peters's conclusion that the unscrambled data possess systemic memory of approximately four years. <=156> n155

b. Correlation Integral Tests

William Brock, David Hsieh, and Blake LeBaron have conducted many different kinds of statistical tests to investigate whether public capital markets are nonlinear systems. <=157> n156 The methodology is a technique called time-delayed embedding, in which, rather than conducting [*577] an anlaysis of time-series data using only the raw series and linear techniques, the data are lagged into a number of series and are then examined using nonlinear techniques, primarily a mathematical measure called the correlation integral. Conceptually, the correlation integral may be seen as a nonlinear counterpart to the correlation coefficient used to detect linear dependence in time-series data. <=158> n157 The correlation integral is a measure of the correlation of data in geometric space where the space can be of any dimension, denoted as m. <=159> n158 For example, a dimension plot can be created by embedding a time series of data [x<t>] of length T in m-dimensional space, <=160> n159 done by forming m vectors using lagged series of the original data, [x<t>]. <=161> n160 The correlation integral is an estimator of the probability that two vectors of the time series of length T are within a specified distance of each other. <=162> n161 The size of the specified distance then is reset at various levels and the correlation integral recalculated for each level. The dimension plot then is drawn by graphing the correlation integrals against the specified distances. <=163> n162 The dimension plot so created then is compared with a dimension plot derived in the same way but using random data as the time series. Substantial differences in the plots are evidence that the time series [x<t>] is not random. [*578]

A richer nonlinear test for the presence of randomness is the recurrence plot, which can be used to detect structural changes in a time series of data [x<t>] over its length T. A recurrence plot is a simple graph in which each axis - call them t and s - represents the time period T. A point (t,s) on the graph is darkened when, as in calculating the correlation integral for the dimension plot, two vectors of the time series of specified lengths are within a specified distance of each other. If [x<t>] is a random series, then the probability of any point being darkened is the same for all possible points. Accordingly, the recurrence plot should have a uniform shade. If it does not - if there are large dark areas or large white areas - then this is evidence that the time series [x<t>] is not random. <=164> n163

Although the dimension plot and the recurrence plot can reveal the absence of randomness and the presence of nonlinear dependence and structure, those phenomena may be attributable to volatility (i.e. variance in security price changes) rather than to some deeper structural phenomenon. <=165> n164 A simple sign-scrambling test can be used to check for this, by reconfiguring the dimension plot as follows. Begin with a time series of data [x<t>] of length T. Then specify a random variable, u<t>, with a value of either +1 or -1, with equal probability. Finally, generate a sign-scrambled series, [y<t>], by specifying the following:

[y<t>] = u<t> * x<t>, t = 1 ... T.

Thus, the sign-scrambled series is simply a haphazard alteration of the signs of the data in the original series. If the probability distribution [*579] of sign price changes is symmetrical, then the distribution of [y<t>] should be the same as that of [x<t>]. Therefore, the estimated correlation integral of [x<t>] should equal the estimated correlation integral of [y<t>]. Thus, the dimension plot of sign-scrambled data [y<t>] should be similar to that of the original data [x<t>].

If not, [x<t>] is asymmetrical and that is not consistent with the explanation that nonlinear dependence of [x<t>] arises solely from volatility. <=166> n165

Brock, Hsieh, and LeBaron applied the foregoing tests to weekly returns on both the CRSP Index (1962-1985) and the S&P Index (1928-1985). <=167> n166 Each of these periods also was subdivided into two periods and studied separately - the CRSP Index for 1962-1974 and 1974-1985, and the S&P Index for 1928-1939 and 1950-1962. Brock, Hsieh, and LeBaron first tested both complete data sets for linear dependence and then removed any linear dependence they found so that they could test for nonlinear dependence exclusively, on the full data sets and on each subperiod. <=168> n167 The results of their tests are as follows.

With respect to the whole period of the CRSP Index, the tests detected nonlinear dependence in that the recurrence plot showed a large amount of data clumping in the lower left corner of the plot, representing the early part of the time series. <=169> n168 The authors could not be sure, however, whether the clumping was due to nonlinearities or to the fact that there was simply less volatility in the CRSP Index in the early part of the series. <=170> n169 On the other hand, the dimension plot for the first subperiod of the CRSP series showed strong nonlinear structure, and the sign-scrambled dimension plot showed asymmetry. <=171> n170 The dimension plot for the second subperiod of the CRSP series showed some nonlinearity but the sign-scrambled dimension plot showed little asymmetry. <=172> n171 Thus, with respect to the CRSP series, the authors concluded that the first subperiod contained nonlinear structure but the second probably did not. <=173> n172

The recurrence plot for the S&P series showed large white spaces in the turbulent period of the early 1930s and, in contrast, large dark spaces in the highly stable period of the early 1960s. <=174> n173 Again, the [*580] dark spaces can signify either a relative absence of volatility or the relative presence of more nonlinear structure. <=175> n174 The dimension plot for the first subperiod of the S&P series was similar to that of the first subperiod of the CRSP series, showing nonlinearity, but the sign-scrambled dimension plot showed little asymmetry. <=176> n175 The dimension plot for the second subperiod of the S&P series showed no strong evidence of nonlinear structure, and the sign-scrambled dimension plot of that data showed no asymmetry. <=177> n176

Brock, Hsieh, and LeBaron interpreted the results of their tests cautiously. <=178> n177 They believed the evidence showed nonlinear structure in stock returns. <=179> n178 They point out, however, that some nonlinearities may be due to volatility or to noise trading of the kind discussed in Part I. <=180> n179 Subject to these cautions, they concluded that there is nonlinear structure in public capital markets. <=181> n180 They further concluded that such markets may behave in accordance with chaos theory. <=182> n181

The results of the foregoing empirical tests and a burgeoning empirical and theoretical literature concerning nonlinear dynamics in public capital markets <=183> n182 leave no doubt that there is nonlinear dependence in financial data. <=184> n183 That certainty has moved theory into practice: Investors are using nonlinear models to develop trading strategies to exploit such discoveries. <=185> n184 Although there are skeptics [*581] at the level of practical application, <=186> n185 from the standpoint of theory, the presence of nonlinear dependence in public capital markets eviscerates the random walk model and makes the ECMH largely meaningless. <=187> n186 Nonlinear models also offer an account of public capital market behavior that goes beyond noise theory by suggesting that what might seem to be irrational behavior may be rational, at least in part. <=188> n187 That public capital markets are nonlinear systems opens up the possibility that public capital markets behave in accordance with chaos theory.

B. Chaos Theory

Chaos theory was popularized by the publication of James Gleick's best-selling book, Chaos, in 1987. <=189> n188 The book was devoted primarily to the exposition of chaos in natural science. The potential role of chaos theory in economics and finance was made prominent by the Santa Fe Institute's publication of a volume in 1988. <=190> n189 Perhaps as a result of these works, chaos theory has become an important and growing field within the study of nonlinear dynamic behavior of economic and financial systems. <=191> n190 Because of the extensive regulation of [*582] such systems, these subjects are becoming an important part of legal scholarship in corporate and securities law. <=192> n191

Through chaos theory, physicists discovered that many phenomena in the universe previously thought to be random (unpredictable, exhibiting no pattern) are not random but exhibit significant pattern. <=193> n192 To oversimplify, chaos theory holds that there is pattern to the seeming randomness of physical events occurring in the universe. <=194> n193 Thus, systems that appear to be stochastic - to involve only random motion or behavior under conventional linear modeling - may be deterministic, or exhibit more complex internal dependence than simple linear modeling reveals.

1. Sensitive Dependence on Initial Conditions

Chaos theory has its roots in the 19th century work of Henri Poincare, a French mathematician and physicist who studied the famous three-body problem. <=195> n194 Newton, using his laws of motion and gravitation, proved that it was possible to calculate accurately the future positions and velocities of two mutually attractive material bodies. <=196> n195 Neither Newton nor anyone since, however, has been able to do so for three or more bodies. This three-body problem reveals itself repeatedly to scientists sending space probes to other planets: they chart a course directed to where the planet will be in its orbit when the probe arrives (not where the planet is upon sending), but midcourse corrections are nevertheless necessary because

Newtonian physics can predict accurately only the interaction of two bodies, not three. <=197> n196 Poincare attributed the three-body problem to nonlinearities inherent in multi-body systems as the result of which "small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter." <=198> n197 This insight, now the unifying core of chaos theory, is known as "sensitive dependence upon initial conditions." <=199> n198 [*583]

To illustrate, envision a billiard table standing empty at state t. <=200> n199 A person then places the eight ball at the middle left pocket on a measured point and rolls it on a measured angle toward the far end of the table. The angle at which the ball strikes the cushion at the far end of the table is measured as well as the angle of the ball's departure from that cushion. As the ball travels the geometry of the table, one can continue the measurements. Assuming no friction, it can be stipulated as a rule of billiard-ball motion that the ball will emerge from impacting a cushion at precisely the angle of approach to that cushion. This stipulation means that we have defined a deterministic system under which the future position of the ball at any time, t<o>, can be forecast perfectly (assuming the actual or average velocity of the ball is known).

Now, however, assume that the initial point from which the ball is thrust is varied by x degrees, some infinitesimally small variation, not observed by or known to the forecaster. The forecaster's predictions of the ball's location after hitting the first one or two cushions may be imprecise - off by some small amount - but the imprecision will be negligible. The amount of error will grow exponentially, however, with each subsequent impact so that by some time, t+n, the forecast will be utterly wrong. Thus, disturbing the measure of the initial position of the ball will cause the ball's movement to appear random and unpredictable, whereas knowing that measure enables precise prediction.

A signal characteristic of chaotic systems is this sensitive dependence on initial conditions; a mechanism for detecting that characteristic is therefore necessary. Before discussing such a mechanism, some background is required concerning the lens through which

chaos is studied - phase-space diagrams and attractors.

2. Phase Space and Strange Attractors <=201> n200

Time-series data traditionally have been plotted using simple Cartesian geometry. For example, to plot a time series of a security's price, price is plotted on the vertical axis and chronological time is plotted [*584] on the horizontal axis. In physics, the usual Cartesian graphs can be turned into more powerful pictures called phase portraits plotted in phase space, a presentation that can depict the range of possibilities for a system. The pendulum is the paradigm for illustrating the differences between Cartesian plots of time-series data and phase portraits of the same data, as well as for introducing the notion of the attractor.

Consider a regularly swinging pendulum driven by mechanical force. It will swing back and forth at a steady speed and not come to rest (unless we withdraw its force). A Cartesian time series plot would show such a pendulum's motion as a wavy up-and-down line whose height remains the same as time passes. <=202> n201 The driven pendulum's portrait in phase space can be envisioned as a rectangle. At any moment in phase space, the pendulum's angle would dictate the location of a point horizontally, and the pendulum's speed would dictate the location of a point vertically. As such, the pendulum's portrait in phase space would form a loop, illustrating the pendulum's continual motion through the same sequence of positions repeatedly. <=203> n202 That continual repetition is described as a limit cycle or a limit cycle attractor because the pendulum (which can be called a system) is attracted to that one and only (limit) cycle. [br n:cht,tphf r3][pp tpn40]Charts Go Here

Now consider a pendulum undriven by permanent mechanical force, having instead been started manually, by lifting it to one end of its orbit and letting it go. Without the permanent mechanical force, the undriven pendulum will swing back and forth, gradually reducing speed and coming closer and closer to rest. A Cartesian time series plot of this undriven pendulum would begin with the same wavy up-and-down line depicting the driven pendulum just described, but this line's height would fall gradually and continuously as the pendulum's speed declines. <=204> n203 This pendulum's portrait in phase space also would begin as if it might form a loop, but, owing to its declining speed, the plot would begin to spiral inward continously as the pendulum slowed down. Correspondingly, the plot would converge to the origin. <=205> n204 The origin in this case is described as a point attractor because the pendulum (or system) is attracted to that one and only point.

There is a third type of attractor, which physicists call the strange attractor. The strange attractor describes a system whose phase portrait will be neither a loop nor a spiraling circle, but rather will show some orbits that appear to be random: They do not repeat and are not periodic. They are, however, limited in range. In other words, the portrait will exist in a finite space but will admit of an infinite number of solutions in that finite space. <=206> n205 [*585]

[SEE FIGURE IN ORIGINAL] [*586]

[SEE FIGURE IN ORIGINAL] [*587]

[SEE FIGURE IN ORIGINAL] [*588]

Limit cycle attractors and point cycle attractors do not exhibit any sensitive dependence on initial conditions: a pendulum without permanent mechanical force will always end up at the point of origin (its point attractor) no matter where it started, and a pendulum with permanent mechanical force will always orbit in its loop (its limit cycle attractor) no matter where it started. Systems containing strange attractors do exhibit sensitive dependence on initial conditions: where the system is at some future moment will be determined by where the system started (or by where it was at any time prior to such future time). Measuring for sensitive dependence on initial conditions can be done by a mathematical operation using what are called Lyapunov Exponents.

3. Lyapunov Exponents

Phase portraits depict all possible states of a system by plotting a variable's value against the possible values of all other variables. The dimension of the phase space is equal to the number of variables that describe the system. Whether a system exhibits sensitive dependence on initial conditions can be determined by numbers called Lyapunov Exponents (LEs). <=207> n206 There is one LE per phase-space dimension (i.e., one LE per variable of the system). <=208> n207 LEs measure the speed of a variable's movements in phase space versus another variable. <=209> n208 Positive LEs measure stretching in phase space - the speed of divergence of one variable with respect to another variable. <=210> n209 Negative LEs measure contracting in phase space - the speed of system restoration after a perturbance. <=211> n210 Thus, LEs for point attractors and limit cycles will never be positive because such systems are always contracting. In the case of a point attractor, the dimensions always converge to a fixed point, the origin; in the case of a limit cycle attractor, all the dimensions converge into one another except one, whose relative position creates the loop by not changing (and whose LE is therefore zero). <=212> n211 For a strange attractor - involving a system that does exhibit sensitive dependence on initial conditions - at least one LE must be positive such that there is divergence in the nearby orbits. <=213> n212 [*589]

4. Chaos and Public Capital Markets

Lyapunov Exponents were created for use in connection with information theory, <=214> n213 to specify the likelihood that information conveyed in binary computer language would be understood properly. <=215> n214 In particular, LEs measured the increase in uncertainty of a communication as additional bits of information were added to the system. The notion of bits of information has been reconceptualized for application to public capital markets as measures of our knowledge of current conditions. <=216> n215 For example, in a time series of security price data (e.g., daily returns), a positive LE would indicate the amount of information or predictive power lost each day. <=217> n216 To illustrate, an LE of .05 per day would mean that information becomes useless after twenty days (i.e., 1/.05). <=218> n217 Thus, the LE is a measure of reliability of information in making forecasts for specified periods. <=219> n218 [*590]

Edgar Peters has calculated the LE for the S&P Index (1950-1989), <=220> n219 using monthly data. <=221> n220 His calculations resulted in a stable LE equal to .0241 per month. <=222> n221 An LE of .0241 per month means that information reliability decays at the rate of .0241 bits of accuracy each month; thus, the average cycle length of the system using this measure is approximately three and a half years (1/.0241 = approximately forty-two months). <=223> n222 Note that this result substantially matches the result that Peters obtained in his R/S

Analysis. <=224> n223 Peters also calculated the LE of the ninety-day trading data for the S&P Index (1928-1990) and found an LE of .09883 per period. That result substantially matches both the monthly LE and the R/S Analysis: the average cycle length of the system was approximately four years (1/.09883 = approximately ten ninety-day periods). Based on these calculations, the public capital markets do exhibit sensitive dependence on initial conditions and chaotic behavior. <=225> n224

5. Fractal Geometry

Another way to test for chaos is to determine whether a system has a fractal dimension. <=226> n225 Systems having fractal dimensions do not follow [*591] Euclidean laws. <=227> n226 Euclidean geometry simplifies and organizes nature dimensionally: there are points, which lack dimension; lines, having one dimension; planes, having two dimensions; and solids, having three dimensions. These simplifying images are heuristics: natural objects do not conform to these images. Until fractal geometry was developed, however, these integral dimensions were all we had to go on.

Fractal geometry was developed initially by Benoit Mandelbrot. <=228> n227 He observed that natural objects are not as simple as the descriptions offered by Euclidean geometry: "Clouds are not spheres [and] mountains are not cones." <=229> n228 For example, how would we classify a piece of paper crumpled up an infinite number of times in terms of Euclidean geometry? It is not three-dimensional because it is not a pure solid form (it has creases and crevices). <=230> n229 It is also not two-dimensional because it has depth. In fact, its dimension is between two and three. That property makes the crumpled paper a fractal: its dimension is a fraction (two-point-something). <=231> n230

With respect to time-series data, dimensionality depends on whether the system from which the data are taken is random or nonrandom. If a system is random, then time-series data taken from it will reflect that randomness and have as large a dimension as can possibly be - usually infinity. <=232> n231 In the case of data being presented on a sheet of paper, the highest possible dimension is two (the dimension of the paper itself). In any case, the data will fill a plane. If a system is nonrandom, then time series of data taken from it will reflect that nonrandomness and will have a fractal dimension: the data will not fill the plane but rather will clump together. <=233> n232 That clumping together reflects the correlations influencing the data (i.e., causing it to be nonrandom). <=234> n233

Edgar Peters also has calculated the correlation integrals for the S&P Index (fractal dimension 2.33), as well as for data from stock [*592] markets in the following countries, which had the fractal dimension specified: Japan (3.05); Germany (2.41); and the United Kingdom (2.94). <=235> n234 Peters also conducted scrambling tests <=236> n235 with respect to all such data and found the following:

If the series is not part of a chaotic attractor, the correlation dimension should not change. If, however, there is a strange attractor present, then scrambling the data should destroy the structure of the attractor, and the correlation dimension should rise. In all cases, the fractal dimension rose, showing that scrambling had a material effect on the analysis.... Thus, we can reject the null hypothesis that there is no chaotic system present. <=237> n236

Accordingly, this test for fractal dimension also suggests that public capital markets exhibit chaotic behavior.

Before applying the insights from the foregoing tests for the presence of nonlinear dynamics or chaotic behavior in public capital markets, a few points must be made clear. The evidence is overwhelming that such markets exhibit nonlinear dependence; that evidence means it is possible, but not necessary, that chaotic behavior is present. Although there is separate evidence showing chaotic behavior, that evidence is by no means definitive. At a minimum, however, the perspective provided by nonlinear dynamics and the insights from the science of chaos theory open up new ways of understanding public capital market behavior.

III. A New Paradigm: Beyond Efficiency and Noise

The presence of nonlinear dependence in public capital markets undercuts the ECMH, which is founded on assertions about the independence of successive security price changes. <=238> n237 Nonlinear dependence is consistent with the informational/fundamental efficiency distinction and noise theory. <=239> n238 That distinction holds that public capital markets may well process and incorporate information swiftly into security prices (and therefore may be informationally efficient), but that such markets are also infected by biases (and therefore are not fundamentally efficient). Noise theory explains the empirical basis for that distinction by holding that market behavior is driven by a substantial psychological or emotional component. <=240> n239

Nonlinear dependence goes beyond noise theory, however, to suggest that there are deeper structural forces that affect public capital market outcomes that cannot be explained by the psychological or emotional perspective of noise theory alone. For example, noise theory cannot explain evidence that the average cycle length of public capital markets is measurable and is equal to approximately four [*593] years. <=241> n240 Deeper structural phenomena, possibily including chaotic dynamics, also must be at work. As such, this nonlinear perspective not only subverts the ECMH but also neutralizes the value of noise theory by pointing to factors beyond behavioral economics to account for market prices deviating materially from underlying asset values. In other words, although noise theory correctly admits that irrational trading exists, the nonlinear/chaotic perspective implies the need to search for other structural factors in addition to irrational trading. Although a full exploration of all such possible forces is not possible in a single law review article, this Part begins the search, using the context of market crashes to launch the inquiry. The insights then are further developed and applied to several current debates in corporate and securities law - circuit breakers, mandatory disclosure rules, mandatory fiduciary obligations, and relational investing. <=242> n241

A. Chaotic Crashes

Consider the October 1987 market crash. The Dow Jones Industrial Average fell by 22.6% on a single day and nearly 33% in the course of one month. <=243> n242 If stock market prices obey the ECMH and accurately reflect information about fundamental asset values, then the body of available information must have changed dramatically in a matter of days and weeks to produce such dramatic change. Many have identified various bits of information to suggest some proportionality between information change in and around mid-October 1987 and the fall in prices at that time. <=244> n243 Most agree, however, that even [*594] cumulatively the identified bits of information would not justify such dramatic price changes. <=245> n244 Because most also agree that such lack of proportionality between information changes and price changes has characterized market swings occurring at other times, <=246> n245 market breaks cannot be explained within the ECMH framework.

Nor can market breaks be completely explained by noise theory. Although noise theory makes the important point that psychological or emotional trading is likely to be a factor, noise theory models explain biased price changes on the grounds that risk-averse arbitrageurs will not correct the effects of such trading. <=247> n246 When psychological and emotional factors are used to explain why markets crash, however, it is at least plausible to believe that there is sufficient room for arbitrage activity that, because of the circumstances, involves less risk than the arbitrageuers concededly would take in the face of everyday noise trading. In sum, neither the ECMH nor noise theory offers a complete explanation of market crashes. <=248> n247

Although no compelling empirical proof yet shows that market crashes constitute chaotic behavior, the intuitive case is powerful indeed. <=249> n248 The intuitive case begins by taking a nonlinear perspective of market time, under which market time expands (speeds up) when trading is heavy and compresses (slows down) when trading is thin. <=250> n249 The speed of market time - called intrinsic time - evidences itself in pricing persistence and pricing discontinuity. <=251> n250 Pricing persistence is described in chaos theory as the Joseph Effect, drawn from the familiar biblical story of Joseph interpreting the Pharaoh's dream to mean [*595] seven years of feast followed by seven years of famine. <=252> n251 The presence of this phenomenon in public capital markets is exhibited by bull markets and bear markets in that identifiable trends emerge and endure for significant time periods. Its presence has been demonstrated in public capital markets through tests of the Hurst exponent and other tests for nonlinear dependence discussed above. <=253> n252

Pricing discontinuity is described in chaos theory as the Noah Effect, taken from the biblical story of the Deluge. <=254> n253 Public capital markets exhibit the Noah Effect in price changes. For example, suppose IBM opens at fifty and closes at thirty. That does not necessarily mean that during some point in the trading day an investor could have traded IBM at forty (or any other price between fifty and thirty). Rather, the price of a security moves discontinuously in the sense that at moment t, it may be trading at forty-five, and at moment t+1, it cannot be sold for more than thirty-five.

The alternating presence of price persistence (the Joseph Effect) and price discontinuity (the Noah Effect) shows that chronological time - a linear concept - is not the most precise temporal measure of public capital market phenomena. <=255> n254 When discontinuous pricing - the Noah Effect - dominates a market, wide swings occur and market pricing is relatively unstable because intrinsic time and trading activity outpace chronological time and information gathering. When price persistence - the Joseph Effect - dominates a market, pricing is relatively stable because intrinsic time is approximately equal to, or slower than, chronological time. On this view, markets swing widely when the speed of intrinsic time accelerates rapidly beyond chronological time: price changes consequently move ahead of information changes. Part of this process may be explained in terms of noise theory to the extent it explains part of why intrinsic time may accelerate in the first place. But noise theory does not explain all of it. For example, once intrinsic time soared ahead of chronological time, it was the structure of the market itself that forged the route to the crash in October 1987, which may be seen as having followed a chaotic route. <=256> n255 [*596]

A more plausible explanation than noise theory is to recognize that investors and other market participants conform perfectly neither to the linear assumption of homogeneous expectations nor to the irrational accusations of noise theory. Rather, investors have heterogeneous expectations that may or may not be rational and that may be defined according to a number of variables. Chief among them are investor time horizons, ranging from the very short-term (for traders such as marketmakers) to the very long-term (for traders such as central banks). <=257> n256 The range of different time dimensions contributes to the Joseph and Noah Effects, persistence, discontinuity, and premature and delayed adjustments to information. Short-term traders react more quickly to new information; long-term investors more slowly. Therefore, information changes will not produce proportionate price changes. Moreover, those changes that are produced constitute new information, producing another round of price changes again defined according to a range of discrete time dimensions. Adding further complexity to this mix of investor heterogeneity and time dimensions is the global nature of financial markets: news itself is dynamic, travelling around the world, usually in twenty-four-hour cycles, and impacting Tokyo, then London, then New York, and around again. <=258> n257

In this reality, it seems implausible to claim instantaneous, unbiased market adjustment to new information, and it is not necessary to attribute all market preadjustment or readjustment to irrational noise trading. <=259> n258 In addition to these complexities concerning investor behavior, existing market structures further complicate market processes at the level of trader actions. <=260> n259 Trader actions are not only - and perhaps not primarily - influenced by the flow of information through the market (whether fundamental, noisy, or otherwise). <=261> n260 Trader actions are influenced strongly by the market environment in which particular traders operate. For example, trader actions are different depending on whether the market is a continuous agency-auction market such as the NYSE, a dealer-market model [*597] like the NASDAQ, or the call-auction models of proprietary trading systems such as Posit, Instinet, and the Arizona Stock Exchange. <=262> n261

Because it is not possible for any party acting alone to determine or set the price of a security, <=263> n262 price discovery in capital markets arises solely as the result of traders' orders meeting in the market. <=264> n263 The importance and complexity of accurate price discovery through this process is critical but often overlooked. <=265> n264 Accuracy of price discovery depends in large part on transparency, the extent to which traders disclose, in real time, their orders, quotes, and trades and the extent to which they make order flows publicly known. <=266> n265 In continuous markets like NASDAQ and the NYSE, transparency is limited, and this interferes with accurate price discovery. <=267> n266 Transparency is further obscured by fragmentation of order flow, both spatially (e.g., where trades are made in different markets) and temporally (e.g., where trades are made continuously). <=268> n267 Moreover, the immediacy of trading offered in continuous markets often contributes to market imperfections through higher trading costs, including the costs of the bid-ask spread, market-impact costs of particular trades, and commissions. <=269> n268

Whether it is necessary, possible, or desirable to minimize the existence or consequences of market opaqueness, fragmentation, and immediacy is beyond the scope of this Article. <=270> n269 Rather, the point is that these systemic complexities reveal additional inadequacies of the ECMH and noise theory and suggest partial explanations for the observed nonlinear dependence of security prices as well as the possible presence of chaotic phenomena. In short, together with the complexities of investor behavior, these market forces and their impact on trader behavior show that the subject of market behavior is far more complex than the current debates over relative efficency and relative rationality would suggest. [*598]

B. Selected Implications

As the foregoing discussion suggests, neither the discovery of nonlinear dependence nor the potential for chaotic behavior in public capital markets will necessarily simplify our understanding of these markets. Both, however, are likely to improve the accuracy of what understanding we have. In particular, a nonlinear perspective allows us to break out of the ECMH/noise theory paradigm and to address market competition and regulation in a more real-world way. This context-breaking understanding also would require (and enable) a comprehensive review of a good deal of corporate law and securities regulation that has been supported by the ECMH and debated at the level of capital market theory only by invoking critiques of the ECMH on its own terms. <=271> n270 The presence in public capital markets of nonlinear dependence and possible chaotic behavior fundamentally alters existing perspectives on all of these debates because it breaks out of the efficiency continuum. <=272> n271

1. Circuit Breakers

In the wake of the 1987 market crash, the major national securities exchanges adopted circuit breakers to forfend future crashes. <=273> n272 Although there are several varieties of circuit breakers, <=274> n273 in general [*599] they trigger when specified price-level changes are reached and then impose a trading halt for a specified period of time - a cooling off period (COP). <=275> n274 It is a truism that market crashes tend to be accompanied by declining security prices. The price-level circuit breakers, however, focus on the effect of the problem rather than its cause. Because no one has been able to explain the causes of the October 1987 market crash, this focus on effects was necessary by default.

Implicit in the price-level circuit breaker is the identification of one perceived cause of market crashes, illiquidity, because the COP is designed to bring buyers back into the market after it is lifted. <=276> n275 Isolating illiquidity as the cause of market crashes poses serious problems, however. First, the meaning of liquidity in public capital markets and how to quantify it have been the subject of long-standing and unresolved discussion. <=277> n276 Second, whatever liquidity means and however it is quantified, the certainty of trading halts upon reaching a specified price level threatens to destroy liquidity by heightening fears and operating as a magnet to pull the market to the trigger level. <=278> n277 Although the COP would restore liquidity by calming fears and bringing buyers back into the market after it is lifted, <=279> n278 the reduction and restoration of liquidity occur at successive moments in chronological time. As such, even to the extent that the price-level circuit breaker implicitly identifies illiquidity as a cause of market crashes, the breaker operates in a linear mode - chronological time - thus ignoring the complexity implied by intrinsic time and the Joseph and Noah [*600] Effects. <=280> n279 Third, isolating and implicitly attempting to regulate relative liquidity is a hazardous enterprise because liquidity is not always desirable, particularly in overly large doses and particularly when driven by regulatory practice. Excess liquidity can produce hyperefficiency - the converse of market crashes - in which because the costs of trading are reduced, the market value of a financial asset tends to increase substantially above the underlying fundamental values it purports to represent. <=281> n280 In such hyperefficient markets, booms feed on themselves, not because of any new fundamental information, <=282> n281 but because the speed of intrinsic time outpaces chronological time. These three points imply the fourth and final one: as nonlinear dynamics and chaos theory suggest, and as everyone knows intuitively, many forces other than relative liquidity affect trading volume and patterns, and therefore any market or regulatory mechanism addressing market crashes must be evaluated in the larger context of such other forces. <=283> n282

The nonlinear perspective and chaos theory, therefore, imply at a minimum that the circuit-breaker analysis should be refocused in terms of nonlinear, intrinsic market time and should focus explicitly on the cause of the problem rather than its effect. Recall that when the market is dominated by the Joseph Effect, intrinsic time is compressed, and when it is dominated by the Noah Effect, it expands. <=284> n283 Computer technology capable of applying nonlinear analysis can monitor the relative dominance of the Noah Effect and the Joseph Effect continually by calculating the probable manner in which different traders will respond to price changes and other events in intrinsic time. <=285> n284 For example, the software can draw a series of graphs approximating how various representative traders will react to a price change [*601] or other event in intrinsic time, with slopes defined according to various time horizons and risk profiles of the representative investors. <=286> n285 The models assume that investors act in a nonlinear fashion: little at first, then more, then little again, all in various proportions. The software then aggregates the models to estimate how the market as a whole will react to price changes or other events, giving a measure of intrinsic time.

According to this view, rather than focusing on negative price-level changes as the existing circuit breakers do, trading halts could be devised based on the speed of intrinsic time (SIT), monitored in accordance with such computer models. When the Noah Effect threatens to dominate the Joseph Effect - when intrinsic time threatens to outpace chronological time such that information gathering lags behind trading activity - a circuit breaker would be triggered. Because SIT is not influenced by the direction or level of price changes, the SIT circuit breaker alleviates the magnetic effect of a negative price-level circuit breaker and also curtails hyperefficiency and its distortions. <=287> n286 Furthermore, rather than halting trading entirely, the SIT circuit breaker should be more modulated; for instance, it would be sufficient to halt new trading orders until some number of existing trading orders have been executed, and then lift the circuit breaker. Coupled with the SIT trigger, modulation would reduce the importance of the COP in easing fears, not only because the magnet effect of existing price-level circuit breakers exaggerates the significance of the COP, but also because contracting the speed of intrinsic time in many cases would render the COP unnecessary - information-gathering in chronological time would catch up with intrinsic time.

It is not my intention to claim that the foregoing prescription concerning circuit breakers will necessarily solve all problems that existing circuit breakers are meant to address. The point is a far more modest one: whatever solutions are designed - whether regulatory, [*602] competitive, or otherwise - should be developed in the context of a broader understanding of market dynamics. <=288> n287

2. Mandatory Disclosure Rules

In an important article confronting the ECMH with noise theory, Professor Langevoort identified the following irony in current securities jurisprudence: although strong claims of market efficiency sometimes suggest that securities regulation is unnecessary, strong claims of market noise suggest that such regulation sometimes is unhelpful and irrelevant. <=289> n288 With respect to mandatory disclosure rules, for example, the ECMH has been invoked to deny the legitimacy of the investor-protection rationale of such rules, <=290> n289 and the noise-theory critique could be read to imply that the information may not be helpful to investors anyway, because they may trade irrationally. <=291> n290 Professor Langevoort's interesting observation relies on an efficiency frame of reference, however, in the sense that noise theory is treated simply as an example of a lack of fundamental efficiency and an excess of informational efficiency.

Chaos theory breaks this mold in two ways. First, as demonstrated extensively above, it holds that public capital market phenomena that seem random (efficient) are in fact nonrandom and internally dependent. Second, chaos theory suggests that some behavior or action that may appear to be irrational (noisy) may well be rational. This is because noise theory attempts to explain price changes that are disproportionate to informational changes. The explanation given is that noise trading - trading not based on fundamental values - persists in keeping market prices divergent from fundamental values because arbitrageurs are not willing to risk correcting the irrational pricing.

A nonlinear perspective and chaos theory suggest, however, that not all actions unrelated to information changes are irrational; instead, such actions may constitute a nonlinear response in a system that itself is inherently nonlinear. As just shown, for example, investor action in the absence of specific news may be rational not only for liquidity and [*603] risk-adjusting reasons, but also because the speed of intrinsic time has accelerated ahead of that of chronological time. In addition, investor overreaction or inaction in the face of specific news (such as improved earnings or a new product announcement) may not be irrational because of different investor time horizons. Such behavior also may not be irrational if broader macroeconomic, technical, or structural factors dictate that optimism or caution should accompany a particular bit of news. Moreover, the behavioral account from noise theory does not address structural factors such as market opaqueness, transparency, fragmentation, and immediacy.

On this view, fundamental information is still critical to investors, but its impact on markets is not to be evaluated by simplified techniques constrained by a linear frame of reference, either as to individual investors or traders or to the market as a whole. The information changes are not discrete in a linear way but are only one contribution - though an indispensible one - to a systemic process that accounts not only for incremental information on its own discrete terms but also for the systemic changes that information implies, all with respect to different time dimensions and structural contexts. In particular, without other informational variables - fundamental, macroeconomic, technical, noisy, or structural - and without acknowledging different time dimensions a simple linear equation such as those used in event studies cannot express the impact of a discrete bit of information on price. With those other variables and multiple time dimensions accounted for, the story is more complex, because these factors all affect price. <=292> n291 Metaphorically, the rules of the game change as the game is played - nonlinear processes are like "walking through a maze whose walls rearrange themselves with each step" taken. <=293> n292

In short, the descriptive paradigms of ECMH efficiency versus noisy inefficiency and of ECMH rationality versus noisy irrationality seem falsely constrained by a linear frame of reference. The system described is forced into a preconceived box defined in terms of relative [*604] efficiency and relative rationality. The nonlinear reality forces the analysis out of this preconceived box and into a multidimensional domain in which notions of rationality and efficiency are only part of the whole story. In the context of mandatory disclosure rules, information currently subject to disclosure remains important to investors, although both its relationship to other information and its reception according to different time horizons and market environments are tremendously important. Unlike the ECMH and noise theory, therefore, chaos theory not only can defend existing mandatory disclosure rules but also implies a possible justification for expanded disclosure of information concerning price discovery in the market microstructure. <=294> n293

3. Mandatory Fiduciary Duties

The methodology of the critique and debate over mandatory disclosure rules in the light of capital market theory has been replicated in the debate over mandatory fiduciary obligations of corporate directors and officers. In that debate, strong claims of market efficiency imply that mandatory fiduciary duties are unnecessary because any risks of shirking, self-dealing, or other opportunism are reflected in market prices. <=295> n294 When noise theory was invoked to challenge this premise and its implication, ECMH devotees offered three responses. First, some said or implied that noise theory was simply too trivial to warrant serious discussion. <=296> n295 Second, some said opponents of the ECMH and the contractarian paradigm of corporate law had the burden of proof of demonstrating, through scientific evidence, that the ECMH and its implications were invalid and that noise theory had not met that requirement. <=297> n296 Third - and most responsive and interesting - Professors Macey and Miller took the position that if public capital markets can be informationally efficient but fundamentally inefficient (i.e., if noise theory shows the ECMH is false), then at its logical conclusion "much of corporate law simply is irrelevant to shareholder welfare." <=298> n297 In particular, according to this argument, if there is a trade-off between these kinds of efficiencies, then investors may prefer that managers deploy resources to promote stock prices - exploiting informational efficiency - instead of allocating those resources to uses [*605] that would promote the value of the underlying assets - maximizing fundamental values. If true, then this theory militates in favor of overruling virtually all of the basic doctrines of corporate law, which obligate corporate management to maximize firm value. These include the fiduciary duties of care and loyalty, the corporate opportunity doctrine, and the director conflict of interest statutes, [and] many other doctrines grounded in the notion that corporate officers and directors have a residual obligation to maximize firm value for shareholders. <=299> n298

As with the mandatory disclosure debate, although strong claims of market efficiency imply that mandatory fiduciary duties are unnecessary, strong claims of market noise imply that those duties are irrelevant (or at least unmanageable). The central insight of chaos theory solves this disturbing irony: Directors and officers cannot control their company's stock price. <=300> n299 Hence, fiduciary duties logically cannot require them to maximize stock price. They can be charged only with managing their own business and its fundamental asset values - not with how the market understands that performance. <=301> n300 Investors and traders make those determinations, <=302> n301 not managers. <=303> n302 [*606]

4. Relational Investing

That corporate managers cannot be charged with maximizing stock prices because they are at the mercy of investors, traders, and the market implies a final point concerning investor-management relations. Professor Roe has observed that the ECMH, coupled with fiduciary obligations of institutional investors, has led such investors to adopt indexing strategies that produce small, inactive shareholders with little incentive to monitor corporate performance. <=304> n303 Those strategies rely on MPT's directive to diversify asset investment <=305> n304 and are faciliated by CAPM's assertion that b can measure security and portfolio risk. <=306> n305 Professor Roe also observed that even those investors who deny the ECMH and trade actively are disabled from active participation in corporate governance and monitoring because of their rapidly changing positions. <=307> n306 Thus, Professor Roe laments that both of these standards - which, he believes, together dominate current investor behavior - advance management goals at the expense of shareholder goals, and that there does not currently exist a theory to justify both large shareholdings and corporate activism. <=308> n307

A nonlinear perspective and chaos theory fill the breach. First, they show that the ECMH is false, although they do not deny that prices impound information. Second, although nonlinear dynamics and chaos theory are not inconsistent with noise theory, they extend beyond noise theory's claim that price distortions are caused primarily by irrational or informationless investing. <=309> n308 As a result, the nonlinear perspective neutralizes concerns that index-based investing will either sustain or exacerbate such distortions. Third, the nonlinear perspective shows that CAPM's b - a linear concept - is not an accurate measure of risk. <=310> n309 Fourth, investors, and not management, are in the best [*607] position to evaluate and translate into market prices the company's performance as measured by its underlying asset values. Of course, investors cannot guarantee translating the story perfectly, not only because of the presence of nonlinear dependence and potentially chaotic behavior in public capital markets but also because of the heterogeneity of investor demand, the pervasiveness of arbitrageurs and short sellers, the role of traders and market structures, the difference between stock and asset markets, and other factors. All of these factors interfere with the identity between asset value and market price. Even subject to this limitation, however, it is worth an investor's time to understand the underlying fundamental values and to promote their identity with market prices. The larger investors have the greater incentives to do this and to reap its rewards, which increase in direct proportion to shareholding size.

Together, therefore, these insights precisely justify a strong boardroom presence to direct the harnessing of fundamental, technical, and structural forces in order to maintain and enhance both fundamental and stock-market values. The nonlinear perspective, however, involves a twist in the usual tale. Rather than asking whether relational investing will minimize or even solve the Berle-Means separation of ownership from control, <=311> n310 the nonlinear perspective permits the hope that relational investing will correct or at least minimize what the ECMH's linear methodology has papered over for nearly two decades: the systemic separation of real asset values from financial asset values. <=312> n311 [*608]

Conclusion

This Article suggests that legal scholars and policymakers focus more closely on recent developments in mathematics and physics that call into question even the weak form of the ECMH. The Article examined the historical roots of the ECMH and the random walk model of public capital market behavior, which the ECMH explains. That history reveals that the random walk model is based on linear mathematical models. Those models have become obsolete, however, as a result of recent advances in the mathematics of nonlinear dynamics and theoretical and applied physics. Those advances indicate that there is nonlinear dependence in the public capital markets, contradicting the random walk model. If the random walk model is not an accurate account of public capital market behavior, then the ECMH is largely meaningless because it answers the wrong question. Although there is also substantial evidence that such markets are noisy, noise theory is an incomplete explanation of public capital market behavior that is itself constrained by the linear perspective that infects the ECMH, leaving the debate focused on relative efficiency.

The scrutiny applied to the ECMH since the October 1987 market crash is important. It is inadequate and incomplete, however, because it is clouded by a lack of inspection of the weak form of the ECMH and by a failure to recognize the limits of noise theory as an explanatory model. The result implies disturbing ironies, suggesting that any analysis of market behavior is aridly binary because either (a) markets are efficient and therefore disclosure rules and fiduciary duties are unnecessary or (b) markets are not efficient but rather are noisy and therefore disclosure rules and fiduciary duties are either unhelpful, irrelevant, or unmanageable.

Chaos theory offers a new perspective for analyzing public capital markets that breaks down the binary perception of markets as either efficient or not efficient. Chaos theory suggests that the problem is not so simple. Rather, it offers insights that provide for a broader vision of both public capital markets and the appropriate nature and degree of rules to regulate them: it suggests that what makes markets inefficient are deeper structural forces than mere noise. As such, it calls for the opening of a new chapter in a broad array of securities and corporate law issues that for nearly two decades have been dominated by linear thought, policy, and practice in American corporate life.

FOOTNOTES:

n1. See, e.g., Thomas L. Hazen, The Short-Term/Long-Term Dichotomy and Investment Theory: Implications for Securities Market Regulation and for Corporate Law, *70 N.C. L. Rev. 137, 158-60 (1991)* (briefly discussing the possible role of chaos theory in the context of efficient capital market hypothesis, the subject of this Article); Henry T.C. Hu, Misunderstood Derivatives: The Causes of Informational Failure and the Promise of Regulatory Incrementalism, *102 Yale L.J. 1457, 1498-99 n.247 (1993)* (briefly discussing chaos theory in the context of the capital asset pricing model, discussed infra notes 94-123 and accompanying text); David G. Litt et al., Politics, Bureaucracies, and Financial Markets: Bank Entry into Commercial Paper Underwriting in the United States and Japan, *139 U. Pa. L. Rev. 369, 450 (1990)* (describing chaos theory as a "new way of looking at dynamic systems" that suggests complexities so great that primary explanations may become unreliable); Glenn H. Reynolds, Chaos and the Court, *91 Colum. L. Rev. 110, 110-11 (1991)* (discussing chaos theory as a metaphor for a theory of Supreme Court jurisprudence); Robert E. Scott, Chaos Theory and the Justice Paradox, *35 Wm. & Mary L. Rev. 329 (1993)* (depicting chaos theory as an heuristic device to break out of an otherwise irreconcilable "Justice Paradox" - a tension between justice in the present case and justice in future cases); Daniel Shaviro, Beyond Public Choice and Public Interest: A Study of the Legislative Process as Illustrated by Tax Legislation in the 1980s, *139 U. Pa. L. Rev. 1, 100 (1990)* (suggesting that the legislative process is an example of a complex system to which chaos theory may be applied); Andrew R. Simmonds et al., Dealing With Anomalies, Confusion and Contradiction in Fraud on the Market Securities Class Actions, *81 Ky. L.J. 123, 144-47* (1992-93) (briefly mentioning the possible role of chaos theory and nonlinear dynamics in capital market theory, the subject of this Article).

n2. See George Melloan, On Stop Lights, Chaos Theory and Russia, Wall St. J., May 3, 1993, at A17 (""Chaos theory' has become one of the hottest fields of science and has begun to penetrate broader public discussion."). Chaos theory also has infiltrated popular culture in the book and the film Jurassic Park; however, like many popular efforts to invoke scientific theory, these failed to capture the essence, and hence the profound paradigmatic implications, of chaos theory. See generally Michael Crichton, Jurassic Park 71-76 (Ballantine Books 1991) (1990) (character, Professor Ian Malcolm, discussing chaos theory's application to prehistoric theme park); Jurassic Park (Universal Pictures 1992).

n3. See, e.g., Ronald J. Gilson & Reinier H. Kraakman, The Mechanisms of Market Efficiency, *70 Va. L. Rev. 549, 549-50 (1984)* (arguing that ECMH has become "the context in which serious discussion of the regulation of financial markets takes place").

n4. Linear means two things in this context: The statistical models themselves attempted to fit straight lines to stock-price data in order to determine whether there was correlation in the data over time, see infra text accompanying notes 23-56; more fundamentally, the motivation for this kind of modelling was the "linear" intuition that there is a proportional relationship between information changes and price changes. See infra text accompanying notes 126-30.

n5. Shelf Registration, Securities Act Release No. 33-6499, *48 Fed. Reg. 52,889, 52,892 (1983)* (codified at 17 C.F.R. pt. 230) (noting that integrated disclosure "recognizes the applicability of the efficient market theory" to publicly traded companies, and relying on the same theory to address disclosure concerns regarding shelf registration); Adoption of Integrated Disclosure Systems, Securities Act Release No. 33-6383, *47 Fed. Reg. 11,380, 11,382 n.9 (1982)* (codified in scattered parts of 17 C.F.R.) (relying on "the premise that information regularly furnished to the marketplace ... may be reflected in the price of the outstanding securities" to support Regulation S-K integrated disclosure provisions); see Barbara A. Banoff, Regulatory Subsidies, Efficient Markets, and Shelf Registration: An Analysis of Rule 415, *70 Va. L. Rev. 135, 138 & n.13 (1984)* (stating that the integrated disclosure system is based in part on a form of ECMH and concluding that "the Commission has ... adopted the theory of efficient markets"); Merritt B. Fox, Shelf Registration, Integrated Disclosure, and Underwriter Due Diligence: An Economic Analysis, *70 Va. L. Rev. 1005, 1008 & n.12 (1984)* (arguing that the SEC relied on the ECMH in permitting shelf registration). But see Donald C. Langevoort, Theories, Assumptions, and Securities Regulation: Market Efficiency Revisited, *140 U. Pa. L. Rev. 851, 856 (1992)* (arguing that although the SEC expressly invoked the ECMH to support integrated disclosure and shelf registration, those regulatory reforms can be justified without regard to the validity of the ECMH).

n6. *Basic Inc. v. Levinson, 485 U.S. 224, 246-47 (1988);* see Ian Ayres, Back to Basics: Regulating How Corporations Speak to the Market, *77 Va. L. Rev. 945 (1991)* (analyzing the validity of the Basic Court's reliance on ECMH in permitting a fraud-on-the-market cause of action); Barbara Black, Fraud on the Market: A Criticism of Dispensing with Reliance Requirements in Certain Open Market Transactions, *62 N.C. L. Rev. 435 (1984)* (analyzing fraud-on-the-market and ECMH); Jonathan R. Macey et al., Lessons from Financial Economics: Materiality, Reliance, and Extending the Reach of Basic v. Levinson, *77 Va. L. Rev. 1017 (1991)* (same). The California Supreme Court, on the other hand, explicitly has rejected an invocation of the ECMH in a state securities-fraud action premised on the fraud-on-the-market theory. See *Mirkin v. Wasserman, 858 P.2d 568, 571-72 (Cal. 1993).*

n7. See, e.g., Homer Kripke, The SEC and Corporate Disclosure: Regulation in Search of a

Purpose 83-139 (1979) (recommending drastic changes in SEC disclosure requirements in the light of ECMH); Joel Seligman, The SEC and the Future of Finance 312 (1985) (predicting or suggesting a number of changes in corporate disclosure, appraisal, and other regulations based on ECMH assumptions); Lucian A. Bebchuk, The Case for Facilitating Competing Tender Offers, *95 Harv. L. Rev. 1028 (1982)* (arguing that, consistent with ECMH, management should be allowed to facilitate competing tender offers); Roger J. Dennis, Materiality and the Efficient Capital Market Model: A Recipe for the Total Mix, *25 Wm. & Mary L. Rev. 373 (1984)* (arguing for a definition of "materiality" based on ECMH teachings); Frank H. Easterbrook & Daniel R. Fischel, Mandatory Disclosure and the Protection of Investors, *70 Va. L. Rev. 669 (1984)* [hereinafter Easterbrook & Fischel, Mandatory Disclosure] (arguing that ECMH makes conventional arguments in favor of mandatory disclosure unpersuasive); Frank H. Easterbrook & Daniel R. Fischel, Corporate Control Transactions, *91 Yale L.J. 698 (1982)* (arguing for a system of allocating gains on sales of control in the light of ECMH); Daniel R. Fischel, Efficient Capital Market Theory, the Market for Corporate Control, and the Regulation of Cash Tender Offers, *57 Tex. L. Rev. 1 (1978)* (arguing that current tender-offer regulation is misguided in the light of ECMH); John H. Langbein & Richard A. Posner, Market Funds and Trust-Investment Law, *1976 Am. B. Found. Res. J. 1* (arguing that ECMH makes the investment of trust funds in index funds "prudent").

n8. See, e.g., Jeffrey N. Gordon & Lewis A. Kornhauser, Efficient Markets, Costly Information, and Securities Research, *60 N.Y.U. L. Rev. 761 (1985)* (cautioning that many recent legal, regulatory, and policy changes in corporate law have been based on ECMH despite insufficient study and consideration of that theory); Louis Lowenstein, Pruning Deadwood in Hostile Takeovers: A Proposal for Legislation, *83 Colum. L. Rev. 249, 254 (1983)* (arguing that the trend against permitting defenses to takeover bids on the theory that "takeovers are a form of natural selection" in an efficient market is misplaced because "the stock market is not nearly as efficient as one would like it to be"); William K.S. Wang, Some Arguments that the Stock Market Is Not Efficient, *19 U.C. Davis L. Rev. 341 (1986).*

n9. This literature has drawn heavily on scholarship in financial economics. See, e.g., Ayres, supra note 6, at 965-68 & nn. 88-92 (discussing the theoretical and empirical shortcomings of the ECMH as identified by financial economists); Carol R. Goforth, The Efficient Capital Market Hypothesis - An Inadequate Justification for the Fraud-on-the-Market Presumption, *27 Wake Forest L. Rev. 895, 901-11 (1992)* (same); Maria O. Hylton, "Socially Responsible" Investing: Doing Good Versus Doing Well in an Inefficient Market, *42 Am. U. L. Rev. 1, 13-27 (1992)* (arguing that the "inefficiency hypothesis" advanced by some financial economists may make socially responsible investing feasible notwithstanding ECMH); Reinier Kraakman, Taking Discounts Seriously: The Implications of "Discounted" Share Prices as an Acquisition Motive, *88 Colum. L. Rev. 891, 898-901 (1988)* (discussing the "market-hypothesis" - the theory advanced by some financial economists that "share prices are sometimes a very poor estimate of the expected value of corporate assets," despite the teachings of ECMH); James Lindgren, Telling Fortunes: Challenging the Efficient Markets Hypothesis by Prediction, 1 S. Cal. Interdisciplinary L.J. 7, 12-18 (1992) (discussing the "inefficient markets theory" and the "masspsychology theory" of financial economists as well as the many studies showing "beat the market" strategies that may effectively refute ECMH); Macey et al., supra note 6, at 1025-28 (arguing that financial economists do not agree that capital markets are efficient). Some recent law review articles use insights from scholarship in financial economics to evaluate related theories, such as capital asset pricing theory, see infra text accompanying notes 94-123. See, e.g., Richard A. Booth, Discounts and Other Mysteries of Corporate Finance, *79 Cal. L. Rev. 1053 (1991)* (arguing that stocks have downward-sloping demand curves, and thus that many of the assumptions of CAPM are inaccurate); Lynn A. Stout, Are Takeover Premiums Really Premiums? Market Price, Fair Value, and Corporate Law, *99 Yale L.J. 1235, 1245 (1990)* [hereinafter Stout, Premiums] ("Financial economists have begun to develop stock pricing models that ... recognize that investors have differing opinions of stock value. Those models provide an account of demand inelasticity ... that differs dramatically from CAPM's."); Lynn A. Stout, The Unimportance of Being Efficient: An Economic Analysis of Stock Market Pricing and Securities Regulation, *87 Mich. L. Rev. 613, 669-74 (1988)* (arguing that market inefficiency, and its corresponding impact on CAPM, will not deter investors from the trading markets). Others have used such insights to evaluate tax policy. See, e.g., James R. Repetti, The Use of Tax Law to Stabilize the Stock Market: The Efficacy of Holding Period Requirements, *8 Va. Tax Rev. 591, 611-30 (1989)* (arguing that capital asset holding period requirements are not advantageous if the market is irrational).

n10. See, e.g., Larry E. Ribstein, The Mandatory Nature of the ALI Code, *61 Geo. Wash. L. Rev. 984, 997-98 (1993);* see also infra text accompanying note 296.

n11. Cf. Arthur J. Jacobson, Autopoietic Law: The New Science of Niklas Luhmann, *87 Mich. L. Rev. 1647, 1657 n.26 (1989)* (reviewing Autopoietic Law: A New Approach to Law and Society (Gunther Teubner ed., 1988)) ("Ideas do not exactly travel at the speed of light in legal theory.").

n12. Unlike legal scholars, financial economists regard scholarship concerning the weak form as the most important part of the current debate. Compare Langevoort, supra note 5, at 853 n.7 ("Most legal writing today focuses on the semi-strong form. Worth noting, however, is that the weak form is still a matter of interest and the subject of continued testing.") with Eugene F. Fama, Efficient Capital Markets: II, 46 J. Fin. 1575, 1609 (1991) ("There is a resurgence of interesting research on the predictability of stock returns from past returns and other variables. Controversy about market efficiency centers largely on this work.").

n13. The meaning of random is discussed infra note 32.

n14. Louis Bachelier, Theory of Speculation, in The Random Character of Stock Market Prices 17 (Paul H. Cootner ed., rev. ed. 1964).

n15. Linear correlation and the term random are discussed infra text accompanying notes 29-35.

n16. Richard A. Brealey & Stewart C. Myers, Principles of Corporate Finance 291 (4th ed. 1991).

n17. Chris Mader & Robert Hagin, The Dow Jones-Irwin Guide To Common Stocks 48 (1976); see also Jeremy Bernstein, Cranks, Quarks, and the Cosmos: Writings on Science 35 (1993) (discussing Brownian motion); Manfred Schroeder, Fractals, Chaos, Power Laws: Minutes from an Infinite Paradise 128 (1991) (same, in context of stock markets). A rigorous exposition of the presence of Brownian motion in public equity markets is M.F.M. Osborne's analysis, Brownian Motion in the Stock Market, in The Random Character of Stock Market Prices, supra note 14, at 100.

n18. Clive W.J. Granger & Oskar Morgenstern, Predictability of Stock Market Prices 76 (1970).

n19. See id. (citing Alfonso de Pietri-Tonelli, La Speculazione di Borsa (1913) (repeatedly quoting Bachelier approvingly); Alfonso de Pietri-Tonelli, La Borsa (1923) (same); Holbrook Working, A Random-Difference Series for Use in the Analysis of Time Series, 29 J. Am. Stat. Ass'n 11 (1934) (reporting discovery of the random walk model but without any reference to Bachelier)); Mader & Hagin, supra note 17, at 49 (citing Eugen Slutzky, The Summation of Random Causes as the Source of Cyclic Processes, 5 Econometrica 105 (1937)); Harry V. Roberts, Stock-Market "Patterns" and Financial Analysis: Methodological Suggestions, in The Random Character of Stock Market Prices, supra note 14, at 7, 8 & n.2 (citing Working, supra).

n20. M.S. Kendall, The Analysis of Economic Time-Series - Part I: Prices, 116 J. Royal Stat. Soc'y 11 (1953); see Stephen F. LeRoy, Efficient Capital Markets and Martingales, 27 J. Econ. Lit. 1583, 1587 (1989) (random walk model "inaugurated in earnest with a major statistical study by" Kendall in 1953); see also Brealey & Myers, supra note 16, at 290 (crediting Kendall with "a startling discovery" that stock price changes are random); Mader & Hagin, supra note 17, at 50 (same).

n21. See Hu, supra note 1, at 1473 n.82. Professor Hu relates the story as follows:

In the 1950's, while rummaging through a library, Leonard Savage of the University of Chicago happened upon a small book by Bachelier published in 1914. He sent postcards to his economist friends asking if they had "ever heard of this guy." Paul Samuelson could not find the book in MIT's library, but did locate and then read a copy of Bachelier's doctoral thesis.

Id. (citing Peter L. Bernstein, Capital Ideas: The Improbable Origins of Modern Wall Street 23 (1992)).

n22. See Granger & Morgenstern, supra note 18, at 77 (citing The Random Character of Stock Market Prices, supra note 14, and the works included therein). One reason these insights may have laid dormant for so long is a major 1937 study concluding that stock prices did move in a predictable way. See Alfred Cowles III & Herbert E. Jones, Some a Posteriori Probabilities in Stock Market Action, 5 Econometrica 280 (1937). This study apparently was widely reported: "The widespread belief that Cowles had silenced the skeptics turned would-be researchers away from the subject for nearly 20 years." Mader & Hagin, supra note 17, at 50. In 1960, Holbrook Working discovered a mistake in this study; Cowles then corrected the mistake and found that the revised study in fact supported the random walk model. Alfred Cowles, A Revision of Previous Conclusions Regarding Stock Price Behavior, 28 Econometrica 909, 909 (1960); see also Granger & Morgenstern, supra note 18, at 76-77 (summarizing the foregoing and citing another early but erroneous analysis that may have obscured Bachelier's insights, Harold T. Davis, The Analysis of Economic Time Series (1941)). Another possible reason is that the insights elaborated from 1900 until the early 1960s could not be harnessed on a large scale until the advent of the computer age and the widespread availability at universities and research foundations of high-speed computers. If this is any part of the reason, imagine the challenges theorists will soon face in the light of the recent availability of advanced computers that are able to conduct nonlinear analysis. See infra notes 131-86 and accompanying text. For another possible explanation, see Hu, supra note 1, at 1473 n.80 (citing Bernstein, supra note 17, at 20) (parenthetically describing Bernstein as "attributing Bachelier's obscurity to mathematical error in [a] subsequent paper and to [the] way his thought defied pigeonholing").

n23. See Granger & Morgenstern, supra note 18, at 77; James H. Lorie et al., The Stock Market: Theories and Evidence 55-63 (2d ed. 1985). Most of the major early work is collected in The Random Character of Stock Market Prices, supra note 14.

n24. Eugene F. Fama, Random Walks in Stock Market Prices, 21 Fin. Analysts J. 55, 56-57 (1965).

n25. Id.

n26. Id. Above-normal returns (and phrases of similar import) as used in this Article are returns exceeding the expected return given a specified level of risk. The usual meaning of expected return is determined by applying the capital asset pricing model, discussed infra text accompanying notes 105-23; see also Ayres, supra note 6, at 960 n.64; Wang, supra note 8, at 349-50 n.26.

n27. See generally Neil A. Weiss & Matthew J. Hassett, Introductory Statistics 622-30 (3d ed. 1991).

n28. See Gordon & Kornhauser, supra note 8, at 848 n.249.

n29. The correlation coefficient is equal to the covariance between the sequences divided by the square root of the product of their variances. See John Daintith & R.D. Nelson, The Penguin Dictionary of Mathematics 76 (1989). Technically, the correlation coefficient so calculated is called the product moment correlation coefficient, which is the most general method; other methods are Spearman's rank correlation coefficient, Kendall's rank correlation coefficient, and the biserial correlation coefficient. Id.

n30. Weiss & Hassett, supra note 27, at 623 fig. 12.17.

n31. Id. To illuminate these technical terms, consider the televised lottery drawings in which winning lottery numbers are determined by selecting numbered balls from a bin containing numerous balls with different numbers painted on them. The auditor retrieves a ball, records its number, and replaces the ball. The auditor does this perhaps three times - each time retrieving, recording, and replacing. This process has the property of statistical independence because the number recorded after any retrieval indicates nothing about the numbers recorded either previously or subsequently. See Gordon & Kornhauser, supra note 8, at 847 n.247. Outside of a controlled context such as a lottery bin (and particularly in the context of time-series data such as securities prices), it is extremely difficult to prove statistically that a series of data has the property of statistical independence. The less restrictive property - that data are uncorrelated - is susceptible to statistical proof and allows for conclusions substantially similar to those that follow from the independence property.

n32. Understanding random to mean either independent or uncorrelated is becoming outmoded. That understanding is motivated by linear thought, which accepts many assumptions about human behavior and organized systems that have come under strict scrutiny in recent years. See infra text accompanying notes 126-30 for a discussion of linear and nonlinear. Modern thought concerning randomness is motivated by a nonlinear perspective and defines randomness as a lack of predictability. In this sense, randomness is defined only as a negative concept - that which cannot be predicted - and may be seen as a description of a state of ignorance. See J. Doyne Farmer & John J. Sidorowich, Can New Approaches to Nonlinear Modeling Improve Economic Forecasts?, in The Economy as an Evolving Complex System 99, 100 (Philip W. Anderson et al. eds., 1988). The authors claim that

the statement that economic time series are random is an empirical one: with a better understanding of the underlying dynamics, better measurements, or the computational power to process a sufficient amount of information, behavior that was previously believed random might become predictable.

Id. In particular, it may be possible to model a system that seems to be affected by an infinite number of variables (or, in statistical terms, degrees of freedom) and therefore seems to be random (as conventionally understood) by identifying and evaluating only those degrees of freedom that are excited by the systemic process and ignoring those that are not invoked by the process:

A more likely cause of randomness in economics is complexity, which we will define here as behavior that involves many irreducible degrees of freedom.... .... A turbulent fluid [by analogy] has an effectively infinite number of possible degrees of freedom, but in reality only a finite number of them are excited ....

Id. at 101. This more complex, nonlinear perspective is developed further infra part II.

n33. See Granger & Morgenstern, supra note 18, at 77-80 (presenting the scope, methodology, and results of the original studies). In technical terms, although there were low but significant levels of correlation, the analyses provided no basis on which to reject the hypothesis that there was no correlation in successive price changes. Lorie et al., supra note 23, at 59 (reporting that studies "uniformly found only insignificant departures from randomness").

n34. See Mader & Hagin, supra note 17, at 51. In addition to the evolving definition of randomness, see supra note 32, it has been noted that authentic devices of chance may not even exist. See Malcolm W. Browne, Coin-Tossing Computers Found to Show Subtle Bias, N.Y. Times, Jan. 12, 1993, at C1. For example, random data generated by a computer or other random number generators are deterministic (although they nonetheless are useful because they tend to mimic conventional randomness reasonably well). See Jose A. Scheinkman & Blake LeBaron, Nonlinear Dynamics and Stock Returns, 62 J. Bus. 311, 318 (1989).

n35. Gordon & Kornhauser, supra note 8, at 848; see also Granger & Morgenstern, supra note 18, at 69.

n36. See Lorie et al., supra note 23, at 60 ("Correlation coefficients have an unfortunate attribute. They may be dominated by a few extreme and unusual observations.").

n37. A run is a statistical term used in time-series analysis and is defined by an absence of directional change in a statistic in the time series. Thus, a new run begins any time the direction changes (i.e. from negative to positive, from positive to negative, or from unchanged either to negative or to positive). See The Random Character of Stock Market Prices, supra note 14, at 15, 81; see also Daintith & Nelson, supra note 29, at 287 (runs also can be specified in other ways, such as runs below or above the median of a series).

n38. Granger & Morgenstern, supra note 18, at 80-81 (citing Eugene F. Fama, The Behavior of Stock Market Prices, 38 J. Bus. 34 (1965)); see also Lorie et al., supra note 23, at 60 (describing Fama's study and his use of run analysis to avoid the inherent limits of autocorrelation tests). The other studies included Cowles & Jones, supra note 22, revised in Cowles, supra note 22 (discovering a slight positive serial correlation in the series, see id. at 912 tbl. 1, 914, which is probably attributable to failure to remove from the analysis trend terms induced by overall, long-run stock market increases resulting from macroeconomic growth over the relevant period); Michael D. Godfrey et al., The Random-Walk Hypothesis of Stock Market Behavior, 14 Kyklos 1 (1964) (using spectral analysis techniques to support independence conclusions); Clive W.J. Granger & Oskar Morgenstern, Spectral Analysis of New York Stock Market Prices, 16 Kyklos 1 (1963) (same).

n39. See Fama, supra note 38, at 90-93.

n40. Lorie et al., supra note 23, at 57-60.

n41. Granger & Morgenstern, supra note 18, at 81.

n42. Sidney S. Alexander, Price Movements in Speculative Markets: Trends or Random Walks, Indus. Mgmt. Rev., May 1961, at 7, reprinted in The Random Character of Stock Market Prices, supra note 14, at 199; see also Mader & Hagin, supra note 17, at 55-56 n.1 (describing generally the concept of filter techniques as efforts to eliminate market "noise" by filtering out some level of fluctuation deemed "insignificant"). Other early efforts to design trading rules that systematically could outperform the market include Niederhoffer & Osborne, 61 J. Am. Stat. Ass'n 897 (finding a number of "trend" factors); Charles C. Ying, Stock Market Prices and Volumes of Sales, 34 Econometrica 676, 676 (1966) (finding a correlation between stock price changes and trading volume).

n43. Alexander, supra note 42, at 214.

n44. For another useful illustration, see Gordon & Kornhauser, supra note 8, at 772-74.

n45. The choice of five percent is for illustrative purposes only. Various percentages may be used. This example is an expansion of a filter-rule strategy proposed by Alexander. See Alexander, supra note 42, at 214.

n46. The gain on sale at time (3) is equal to x, the excess of the peak price over the buy price at time (1), minus y, the excess of the peak price over the sale price at time (3).

n47. Going short means borrowing a security and selling it at the prevailing price, promising to repay with the same security, which will be purchased for the price prevailing at the time of repayment.

n48. Covering the short position means repaying the borrowed security by buying at the price prevailing at the time of repayment.

n49. The gain on covering the short position is equal to the excess of the sale (short) price at time (3) over the price prevailing at time (4).

n50. See Alexander, supra note 42, at 214-18.

n51. Sidney S. Alexander, Price Movements in Speculative Markets: Trends or Random Walks, No. 2, in The Random Character of Stock Market Prices, supra note 14, at 338, 340-44.

n52. See Lorie et al., supra note 23, at 60-63 (noting that the "major shortcomings" of

Alexander's initial study were a "failure to realize that dividends were a cost rather than a benefit when stocks were sold short, the failure to take transaction costs into account, and the assumption that stocks could be bought or sold at the precise price when the signal to buy or sell was given"); see also Granger & Morgenstern, supra note 18, at 82 (citing studies showing errors in Alexander's analysis, including Fama, supra note 38, at 81-85 (noting that even in his second work, Alexander failed to account for the fact that the borrower of short stocks must pay the lender any dividends paid while in short position); Eugene F. Fama & Marshall E. Blume, Filter Rules and Stock-Market Trading, 39 J. Bus. 226, 238-41 (1966) (demonstrating that because of transaction costs and brokerage commissions, filter techniques are no more profitable than buy-and-hold practices); Benoit Mandelbrot, The Variation of Certain Speculative Prices, in The Random Character of Stock Market Prices, supra note 14, at 307, 330-31 (observing the inaccuracy of Alexander's assumption that buy orders at five percent will result in purchases at five percent; in fact, investors "will almost always have paid more than assumed" in Alexander's analysis)).

n53. For a thorough discusssion of several of the more conventional technical methods (The Dow Theory, The Relative-Strength System, Price-Volume Systems, Reading Chart Patterns) as well as several of the more unconventional (The Hemline Indicator, The Super-Bowl Indicator, and the Odd-Lot Theory), see Burton G. Malkiel, A Random Walk Down Wall Street 139-48 (5th ed. 1990).

n54. More recent investigations of trading rules again have suggested the possibility of exploiting trends masked by the "jiggling of the market." See, e.g., Guy Charest, Split Information, Stock Returns and Market Efficiency - I, 6 J. Fin. Econ. 265, 292 (1978); Guy Charest, Dividend Information, Stock Returns and Market Efficiency - II, 6 J. Fin. 297, 326-27 (1978). Professor Lindgren has specified several trading rules as part of a longitudinal study he recently launched in which he specifies the parameters and time periods of his study prospectively, the results to be analyzed and published in 2001 and 2016. Lindgren, supra note 9, at 37.

n55. See Granger & Morgenstern, supra note 18, at 83 ("The main point of the [random walk] model is that it says that past prices do not contain (linear) information useful in predicting future price changes.") (emphasis added); id. at 72 ("The correlation coefficient is only a measure of the degree to which two random variables are linearly related."); id. at 84 ("The tests of the random walk hypothesis have necessarily concentrated on linear relationships between a price change and its predecessors. The tests have not excluded all possible types of nonlinear relationships.").

n56. See infra notes 248-87 and accompanying text.

n57. See, e.g., Brealey & Myers, supra note 16, at 290-93; Edwin J. Elton & Martin J. Gruber, Modern Portfolio Theory and Investment Analysis 403 (4th ed. 1991); Macey et al., supra note 6, at 1027-28 n.34.

n58. Paul A. Samuelson, Proof that Properly Anticipated Prices Fluctuate Randomly, Indus. Mgmt. Rev., Spring 1965, at 41. Samuelson won a Nobel Prize in economics in 1970 - the first American to be so honored. The "proof first/theory later" syndrome has continued to plague the ECMH itself. See Gilson & Kraakman, supra note 3, at 551-52 (quoting Professor Beaver as saying the empirical findings on efficient market research "have largely preceded a formal, conceptual development of market efficiency" and concluding therefore that "legal users of the ECMH literature have been, by and large, confronted with a body of empirical evidence in search of a causative theory").

n59. E.g., Paul A. Samuelson, Economics 43, 56 (10th ed. 1976).

n60. See LeRoy, supra note 20, at 1587 (citing Roberts, supra note 19, and stating that Roberts "pointed out that in the economist's idealized market of rational individuals one would expect exactly the instantaneous adjustment of prices to new information that the random walk model implies"). Public capital markets may or may not have the characteristics assumed by the perfect market model. The cardinal rule of economic forecasting, however, holds that a model's predictive power is the only relevant test of its validity, not the assumptions underlying it. See Milton Friedman, The Methodology of Positive Economics, in Essays in Positive Economics 3, 23 (1953) (stating that a "theory cannot be tested by the "realism' of its "assumptions'"); see also Mark Blaug, The Methodology of Economics: Or How Economists Explain 104 (6th ed. 1985) (explaining Friedman's thesis to be that the realism of the assumptions underlying a theory is irrelevant and that models are to be judged by their predictive power). Thus, the fact that investors are not rational or fully informed, for example, is not material, so long as these realities do not interfere with the predictive power of the ECMH. In the light of the waning predictive power of the ECMH, many economists and others are reconsidering such assumptions, particularly the rational-actor assumption. See Lindgren, supra note 9, at 16-18 (comprehensive listing of "anomalies" - empirically documented realities that the ECMH cannot explain). For an excellent critique of the theoretical perfect market model, proving that perfect markets must ultimately be founded on imperfect markets, see David G. Carlson, On the Margins of Microeconomics, in Deconstruction and the Possibility of Justice 265 (Drucilla Cornell et al. eds., 1992); David G. Carlson, On the Margins of Microeconomics, *14 Cardozo L. Rev. 1867 (1993).*

n61. Although Paul Samuelson introduced the ECMH in 1965, the three forms of the ECMH were not introduced until 1970, when Eugene Fama published his classic work surveying efficient market scholarship. Eugene F. Fama, Efficient Capital Markets: A Review of Theory and Empirical Work, 25 J. Fin. 383, 383 & n.1 (1970) (crediting Harry Roberts with the idea for the three forms).

n62. See id. at 383.

n63. See Gilson & Kraakman, supra note 3, at 555-57.

n64. Professor Fama recently has proposed to modify this three-part categorization slightly: instead of the "semi-strong form" tests he would use the term event studies to evaluate the adjustment of prices to public announcements; instead of "strong-form" tests he would use the term tests for private information; and instead of "weak-form" tests, this category would be renamed and broadened to include all tests for return predictability (thus including, in addition to tests based on past returns, tests based on other variables such as dividends and interest rates). See Fama, supra note 12, at 1576-77.

n65. Michael C. Jensen, Some Anomalous Evidence Regarding Market Efficiency, 6 J. Fin. Econ. 95, 96 (1978).

n66. "Direct" link may overstate the case, although some have said that the weak form of the ECMH "springs from the realization that security prices in one period are independent of their prices in other periods, that they follow a "random walk."' Macey et al., supra note 6, at 1024 n.24 (emphasis added because of the argument, infra part II, that the "realization" is false); see also Financial Accounting Standards Board, The Efficient Market Hypothesis (1976), reprinted in James D. Cox, Financial Information, Accounting, and the Law: Cases and Materials 179, 180 n.* (1980) ("A "random walk' process is one in which successive price changes are statistically independent (serial correlation is zero) so that past price changes cannot be used to predict future price changes. Although to do so somewhat oversimplifies some highly technical matters, the random walk model is commonly identified with the so-called weak form of the efficient market hypothesis.").

n67. See supra notes 29-32 and accompanying text.

n68. See, e.g., Gordon & Kornhauser, supra note 8, at 847 n.246.

n69. See supra notes 27-40 and accompanying text.

n70. See Lorie et al., supra note 23, at 63 (stock prices are an indicator of intrinsic values that can change only as a result of information: if there is only a gradual awareness of new information, then successive price changes will exhibit dependence; however, if awareness is instantaneous, then successive price changes will be random); see also Cox, supra note 66, at 183-84. Professor Cox explains:

To document the theorem that stock prices are random because publicly available information changes daily, requires an association of stock prices and information to be established. A finding that stock price changes are associated with the release of new information and that such response occurs rapidly is consistent with the conditions that should be present if security markets are efficient in the semi-strong form. The only way the empiricist can determine whether securities markets are efficient in the semi-strong form is to investigate whether conditions consistent with a market which is efficient in the semi-strong form exist.

Id.

n71. See Christopher P. Saari, Note, The Efficient Capital Market Hypothesis, Economic Theory and the Regulation of the Securities Industry, *29 Stan. L. Rev. 1031, 1040 (1977).* The author observed:

Random movements of security prices, a second characteristic of market efficiency, is similarly essential to the ECMH. If security prices did not move randomly, an investor aware of systematic price movements could use that information to generate above average returns through propitious trading.

Id. (emphasis added).

n72. Many have observed that anyone who knows how to beat the market has virtually no incentive to make that knowledge public and has strong incentives to keep it private; some have observed the opposite. Malkiel, supra note 53, at 187 (speculating that a good many academics have challenged the ECMH in the hopes of publishing dazzling articles that lead directly to tenure). Many also have observed that if the ECMH is valid, then any systematic contrarian trading strategy could not be sustained for long because it would be discovered. E.g., Langevoort, supra note 5, at 871 n.64. The more important point is one that is often overlooked: if the ECMH is valid, then no systematic contrarian trading strategy can exist to be discovered.

n73. See F.H. Buckley, When the Medium Is the Message: Corporate Buybacks as Signals, *65 Ind. L.J. 493, 528* (1989-90) ("The strong version of the ECMH is now taken as disproven insofar as it predicts that insider trading is merely a fair game.") See generally James B. Stewart, Den of Thieves (1991) (a chronicling of the insider-trading scandals of the 1980s, for which Stewart won a Pulitzer Prize).

n74. See, e.g., Jerome B. Baesel & Garry R. Stein, The Value of Information: Inferences from the Profitability of Insider Trading, 14 J. Fin. & Quantitative Analysis 553, 567-69 (1979) (demonstrating that corporate insiders do achieve above-market returns); Daniel W. Collins, SEC Product-Line Reporting and Market Efficiency, 2 J. Fin. Econ. 125, 155-57 (1975) (same); Jeffrey F. Jaffe, Special Information and Insider Trading, 47 J. Bus. 410, 427-28 (1974) (same); James H. Lorie & Victor Niederhoffer, Predictive and Statistical Properties of Insider Trading, *11 J.L. & Econ. 35, 52-53 (1968)* (same); Niederhoffer & Osborne, supra note 42, at 904-08 (explaining how exchange specialists can generate above-normal returns by "contra-tick" trading); see also Cox, supra note 66, at 186 ("Preliminary indications are that securities markets are not efficient in the strong form."); Malkiel, supra note 53, at 198 ("The strongest form of the efficient-market hypothesis is unlikely to hold. We know that stock splits, dividend increases, and merger announcements can have substantial effects on share prices. Consequently, insiders trading on such information can clearly profit before the announcement is made.").

n75. For example, in a widely noted study, Andrew Lo and A. Craig MacKinlay demonstrated strong positive serial correlation in stock prices for weekly and monthly holding-period returns. Andrew W. Lo & A. Craig MacKinlay, Stock Market Prices Do Not Follow Random Walks: Evidence from A Simple Specification Test, 1 Rev. Fin. Stud. 41, 42 (1988) (using 1,216 weekly stock return observations from 1962 to 1985 and finding weekly autocorrelation coefficient of 30%, an extremely high coefficient). Lo and MacKinlay point out that their study does not mean necessarily that the stock market is inefficient but rather that the random walk model cannot be the basis for any theory of efficiency. Id. at 42-43, 61 ("Any structural paradigm of rational price formation must now be able to explain this pattern of serial correlation present in weekly data."); see also Jennifer Conrad & Gautam Kaul, Time-Variation in Expected Returns, 61 J. Bus. 409 (1988) (addressing the criticism of the Lo-MacKinlay study that because the portfolio studied included small capitalization stocks that trade less frequently than larger capitalization stocks, the serial correlation may have been "induced" by the fact that the market absorbs information about large stocks first and thus that there is a lag for information absorption in small stocks). Studies like these have led Eugene Fama, a chief architect of the ECMH, to conclude that daily and weekly stock returns are predictable from past returns, thus rejecting the random walk model on a statistical basis. Fama, supra note 12, at 1580 ("The work thus rejects the old market efficiency-constant expected returns model on a statistical basis."). Professor Fama regards these conclusions as undramatic, however, saying they "tend to confirm the conclusion of the early work that, at least for individual stocks, variation in daily and weekly expected returns is a small part of the variance of returns." Id.; see also James M. Poterba & Lawrence H. Summers, Mean Reversion in Stock Prices: Evidence and Implications, 22 J. Fin. Econ. 27, 53 (1988) (finding both positive serial correlation over short periods and negative serial correlation over longer periods in monthly data (from 1926-1985) and annual data (from 1871-1985) on New York Stock Exchange stock returns, and reporting that although their study "does not consistently permit rejection of the random-walk hypothesis at high significance levels, the various data sets together strengthen the case against its validity"). Professor Fama has agreed with the conclusions Poterba and Summers reach and reported similar studies; nonetheless, he concludes that they provide only "weak statistical evidence against the hypothesis that returns have no autocorrelation and prices are random walks." Fama, supra note 12, at 1581.

n76. See infra notes 124-87 and accompanying text.

n77. See Peter Fortune, Stock Market Efficiency - An Autopsy?, New Eng. Econ. Rev., Mar.-Apr. 1991, at 17, 21-30; Lindgren, supra note 9, at 15-18 (comprehensive summary of the anomalies with respect to the semi-strong form of the ECMH). The numerous documented anomalies at first seem astonishingly broad in aggregate scope and depth. This may be due at least in part, however, to selection bias in the testing process. See LeRoy, supra note 20, at 1610 ("The published literature is skewed toward interesting, that is anomalous, results, and away from boring confirmations of the absence of anomaly."). It also may be due to the joint-hypothesis problem, discussed infra notes 109-16 and accompanying text.

n78. James Tobin, On the Efficiency of the Financial System, Lloyds Bank Rev., July 1984, at 1. Tobin won a Nobel Prize in economics in 1981.

n79. See John M. Keynes, The General Theory of Employment Interest and Money 156 (1936). The distinction also has been noted often by other economists, including other Nobel Prize winners. See, e.g., William F. Sharpe, Portfolio Theory and Capital Markets 104-13 (1970) (Sharpe won a Nobel Prize in economics in 1990); Kenneth J. Arrow, Risk Perception in Psychology and Economics, 20 Econ. Inquiry 1, 5-8 (1982) (Arrow won a Nobel Prize in economics in 1972). More recently, several empirical studies have been conducted that support the informational/fundamental distinction. See, e.g., David M. Cutler et al., What Moves Stock Prices?, J. Portfolio Mgmt., Spring 1989, at 4; Poterba & Summers, supra note 75, at 53-54; Lawrence H. Summers, Does the Stock Market Rationally Reflect Fundamental Values?, 41 J. Fin. 591, 598, 600 (1986); see also Mark Rubinstein, Securities Market Efficiency in an Arrow-Debreu Economy, *65 Am. Econ. Rev. 812, 820-23 (1975)* (distinguishing "informational efficiency" from "intrinsic value").

n80. Explication of the informational/fundamental distinction may result in a division much more complicated than a simple bifurcation of the semi-strong form of the ECMH. See Ayres, supra note 6, at 968-75 (elaborating a matrix of possible classifications).

n81. See supra note 53.

n82. Many legal scholars have noted the distinction between informational and fundamental efficiency but have used different terminology. E.g., Ayres, supra note 6, at 965-75 (informational and fundamental efficiency); Daniel R. Fischel, Efficient Capital Markets, the Crash, and the Fraud on the Market Theory, *74 Cornell L. Rev. 907, 912-13 (1989)* (trading-rule and value efficiency); Merritt B. Fox, The Role of the Market Model in Corporate Law Analysis: A Comment on Weiss and White, *76 Cal. L. Rev. 1015, 1033 n.47 (1988)* (speculative or information-arbitrage and allocative or fundamental-valuation efficiency); Gordon & Kornhauser, supra note 8, at 825 (speculative and allocative efficiency); Wang, supra note 8, at 344-49 (information-arbitrage and fundamental-valuation efficiency); see also Victor Brudney & William W. Bratton, Brudney and Chirelstein's Cases and Materials on Corporate Finance 130-31 (4th ed. 1993) (summarizing the distinction and its underpinnings). Professors Gordon and Kornhauser offer the most formal account of the theoretical basis of this distinction by specifying that speculative efficiency obtains in a market where information concerning expected returns on financial assets is reflected in prices of those financial assets while allocative efficiency exists only if the managers of the productive (or real) assets underlying those financial assets are fully informed with respect to the expected returns on those real assets. See Gordon & Kornhauser, supra note 8, at 767-69. The authors state that the ECMH claims only speculative and not allocative efficiency. Id. at 771, 827.

n83. Some scholars have denied the validity of the distinction between informational and fundamental efficiency. See Jonathan R. Macey & Geoffrey P. Miller, The Fraud-on-the-Market Theory Revisited, *77 Va. L. Rev. 1001, 1012-15 (1991).*

n84. In the context of the ECMH, rational behavior need not obtain at the individual level provided it obtains in the aggregate - provided the result of the process is as if individuals had behaved rationally. In other words, under the ECMH, participation of many investors will produce the outcome that would be produced if all were rational, because the mistakes made by those who do not act rationally will be exploited and thus corrected by those who do. This subject has been analyzed repeatedly for over a century, beginning with Charles MacKay's classic exposition, Memoirs of Extraordinary Popular Delusions (London, Richard Bentley 1841).

n85. The ECMH is based on the same assumptions that underly the perfect market model. See supra text accompanying notes 59-60.

n86. See LeRoy, supra note 20, at 1612 ("Fischer Black came to the rescue. By renaming irrational trading "noise trading' Black avoided the I-word, thereby sanitizing irrationality and rendering it palatable to many analysts who in other settings would not be receptive to such a specification."). Note that noise in statistics (and in communications engineering) means random disturbance, which in turn means error; that usage is distinguished from random in the sense of the random walk, which means independent or uncorrelated. Compare, e.g., Daintith & Nelson, supra note 29, at 229 (definition of noise) with id. at 273 (definition of random walk).

n87. E.g., Robert J. Shiller, Market Volatility 1-48 (1989); Ayres, supra note 6, at 997; William W. Bratton, The Economic Structure of the Post-Contractual Corporation, *87 Nw. U. L. Rev. 180, 203-05 (1992);* Kraakman, supra note 9, at 899-900; Langevoort, supra note 5, at 852-53; Andrei Shleifer & Lawrence H. Summers, The Noise Trader Approach to Finance, J. Econ. Persp., Spring 1990, at 19, 19-20; Joseph E. Stiglitz, Symposium on Bubbles, J. Econ. Persp., Spring 1990, at 13; Wang, supra note 8; see also Amos Tversky & Daniel Kahneman, Rational Choice and the Framing of Decisions, in Decision Making: Descriptive, Normative, and Prescriptive Interactions 167, 167 (David E. Bell et al. eds., 1988) (providing a statistical critique of the presumption of rational choice generally).

n88. E.g., Fischer Black, Noise, 41 J. Fin. 529, 529 (1986).

n89. For a good summary of this literature and a thorough analysis of its relationship to selected securities-law issues (integrated disclosure, shelf registration, and fraud-on-the-market theory), see Langevoort, supra note 5.

n90. See J. Bradford De Long et al., Noise Trader Risk in Financial Markets, 98 J. Pol. Econ. 703, 703-04 (1990).

n91. See id. at 705.

n92. See Kraakman, supra note 9, at 901 n.31 ("Note that noisy or biased share prices are consistent with a broader perspective on market efficiency that looks to costs and returns of acquiring and processing information."); Langevoort, supra note 5, at 852-53 (explaining how economists minimize the impact of noise theory on the ECMH).

n93. The link between noise theory and nonlinear modeling has been suggested informally by Lawrence Summers at a conference concerning nonlinear dynamics and public capital markets sponsored by the Santa Fe Institute in 1988. See Kenneth J. Arrow et al., Report of Work Group B: Economic Cycles, in The Economy as an Evolving Complex System, supra note 32, at 247, 248. Professor Arrow explains:

Another topic which was discussed was to find a simple toy model explaining fluctuations in the stock market in terms of nonlinear dynamics. The idea proposed by Larry Summers is that there are "noisy traders" and "sophisticated traders" which share the market. They have different (deterministic) strategies, and one obtains in this manner rather easily a chaotic time evolution. But it must be admitted that a really convincing model has not yet been obtained.

Id.

n94. See Fama, supra note 12, at 1576 (stating that the theme of his 1970 survey, see Fama, supra note 61, was "that we can only test whether information is properly reflected in prices in the context of a pricing model that defines the meaning of "properly""); see also Menachem Brenner, The Sensitivity of the Efficient Market Hypothesis to Alternative Specifications of the Market Model, 34 J. Fin. 915, 915 (1979) (observing that "to test whether all available information is reflected in market prices we require a model that describes how this information is reflected in prices").

n95. See Fama, supra note 12, at 1589 ("Depending on the emphasis desired, one can say that efficiency must be tested conditional on an asset-pricing model or that asset-pricing models are tested conditional on efficiency. The point is that such tests are always joint evidence on efficiency and an asset-pricing model."); Langevoort, supra note 5, at 851 n.1; see also Richard Roll, A Critique of the Asset Pricing Theory's Tests Part I: On Past and Potential Testability of the Theory, 4 J. Fin. Econ. 129 (1977) (observing that it is nearly impossible to test the validity of asset-pricing theories).

n96. Harry M. Markowitz, Portfolio Selection: Efficient Diversification of Investments (1959). Markowitz won a Nobel Prize in economics in 1990.

n97. See id. at 6.

n98. Mathematically, risk is the variance of possible returns around the expected return. See infra note 136 (defining variance).

N99. For the mathematical steps necessary to calculate the risk of a portfolio, see Brealey & Myers, supra note 16, at 139-43.

n100. See Gordon & Kornhauser, supra note 8, at 777.

n101. See id.; see also Brealey & Myers, supra note 16, at 145-48.

n102. Systematic risk is also called market risk or undiversifiable risk. Brealey & Myers, supra note 16, at 137 n.14.

n103. Unsystematic risk is also called unique risk, residual risk, specific risk, or diversifiable risk. Id. at 137 n.13.

n104. See, e.g., Brealey & Myers, supra note 16, at 137-38; Gordon & Kornhauser, supra note 8, at 778.

n105. See Fama, supra note 12, at 1589.

n106. The CAPM is the most widely used and best-known of the asset pricing models, although there are others. For an excellent discussion of various asset pricing models, see Gordon & Kornhauser, supra note 8, at 847-49 (constant expected returns model), 834-37 (market model), 772-75 (positive expected returns model), 775-86 (extensive discussion and analysis of one-factor and multifactor asset pricing models); see also Robert A. Haugen, Modern Investment Theory 196-258 (3d ed. 1993) (thorough discussion and analysis of fundamentals, written for an introductory graduate or intermediate undergraduate course in investment management); Fama, supra note 12, at 1589-99 (historical and technical discussion of evolution of early pricing models); LeRoy, supra note 20, at 1588-92 (same).

n107. See supra notes 96-104 and accompanying text.

n108. The existence of a risk-free rate of return is assumed by the CAPM: it assumes that everyone can borrow and lend at a risk-free rate (usually taken to mean the 90-day U.S. Treasury bill rate; risk-free is used in the limited sense of principal repayment but does not account, for example, for interest-rate or foreign-exchange risks). More refined versions of capital asset pricing models provide for the possibility that risk-free borrowing is not available. E.g., Fischer Black, Capital Market Equilibrium with Restricted Borrowing, 45 J. Bus. 444, 444, 446, 455 (1972); Fischer Black et al., The Capital Asset Pricing Model: Some Empirical Tests, in Studies in the Theory of Capital Markets 98, 98, 100, 112 (Michael C. Jensen ed., 1972).

n109. Brealey & Myers, supra note 16, at 162.

n110. See supra note 95.

n111. See supra note 77; see also Lindgren, supra note 9, at 16-18 (comprehensive summary of anomalies that the semi-strong form of the ECMH cannot explain).

n112. E.g., Fama, supra note 12, at 1589; Gordon & Kornhauser, supra note 8, at 775-86.

n113. See Gordon & Kornhauser, supra note 8, at 783-84.

n114. See infra text accompanying note 126.

n115. See Gordon & Kornhauser, supra note 8, at 781 n.49.

n116. See supra note 84.

n117. See infra note 155.

n118. See Stout, Premiums, supra note 9, at 1238.

n119. See infra notes 256-58 and accompanying text.

n120. Stout, Premiums, supra note 9, at 1238; Geert Bekaert & Robert J. Hodrick, Characterizing Predictable Components in Excess Returns on Equity and Foreign Exchange Markets, 47 J. Fin. 467, 469 (1992) ("Variation over time in expected returns poses a challenge for asset pricing theory because it requires an explicitly dynamic theory in contrast to the traditional static capital asset pricing model."). The role of rational expectations (or their absence) also has been an important element in the extensive work conducted over the past several years demonstrating excess volatility in public capital markets. See, e.g., Shiller, supra note 87, at 9-63 (developing models and conducting data analysis in the light of evidence of fads, fashions, and bubbles in financial markets); Arrow et al., supra note 93, at 248. Professor Arrow writes:

Larry Summers proposed an explanation [for the connection between speculation and volume, and the problem of excess volatility] that the influx of information is the driving force. Information is interpreted differently by different traders, leading to an increased trading of shares, and an increased volatility. Note that the stock market is itself a source of information; the process is thus self-sustaining.

Id.

n121. E.g., Andrei Shleifer, Do Demand Curves for Stocks Slope Down?, 41 J. Fin. 579 (1986) (especially insightful analysis of the implications of heterogeneous investor demand); Stout, Premiums, supra note 9, at 1239.

n122. In more words, "humans are incredibly changeable, fickle, capricious, faddish, fashion-conscious, moody and given to group or herd psychology." Simmonds et al., supra note 1, at 143.

n123. See supra notes 14-17 and accompanying text.

n124. See supra notes 3-8 and accompanying text.

n125. See supra notes 23-40 and accompanying text.

n126. Brealey & Myers, supra note 16, at 161-62; see also supra notes 106-22 and accompanying text.

n127. Weiss & Hassett, supra note 27, at 623 fig. 12.17; see also supra notes 23-35 and accompanying text.

n128. This is a generalized statement of the distinction between linear and nonlinear, which as a matter of pure science is significantly more complex. See Gregory L. Baker & Jerry P. Gollub, Chaotic Dynamics: An Introduction 1 (1990) (measurement error in nonchaotic systems amplifies linearly over time; in chaotic systems, measurement error amplifies exponentially over time).

n129. See infra note 243.

n130. See supra notes 78-93 and accompanying text. Studies showing excess volatility also reflect a lack of proportionality, indicating a nonlinear process. See supra note 120.

n131. One reason such techniques were unavailable in the 1960s and 1970s was, of course, the need for powerful computer systems that not only could process data more swiftly but also could go beyond the simplified mathematical models of straight lines and investigate curvatures of multidimensional data streams. See Lewis Knox, A Not-So-Random Walk on the Quant Frontier, Institutional Investor, Apr. 1991, at 33.

n132. Melvin J. Hinich & Douglas M. Patterson, Evidence of Nonlinearity in Daily Stock Returns, 3 J. Bus. & Econ. Stat. 69 (1985).

n133. The literature on nonlinear dynamics and chaos theory is voluminous, and much of it is extremely technical - although some of the work is becoming more accessible. I draw heavily on two primary sources that evaluate the applicability of nonlinear dynamics and chaos theory to public capital markets: one by a practitioner who is rather more optimistic about such applicability, Edgar E. Peters, Chaos and Order in the Capital Markets (1991), and one by three academics who are rather less sanguine about such applicability, William A. Brock et al., Nonlinear Dynamics, Chaos, and Instability: Statistical Theory and Economic Evidence (1991). Despite their differences in ultimate outlook, the evidence they introduce and the methodology they use are consistent. They also both point unambiguously to the conclusion that public capital markets are nonlinear systems. They differ in their level of confidence that such markets behave in accordance with chaos theory. In all cases, the literature discussing this subject is unified by open minds and characterized by the phenomenon that as we learn more, we learn that we know even less than we thought.

n134. See Peters, supra note 133, at 62-63.

n135. The range of the observations is the difference between the maximum and minimum cumulative deviations of the observations from the average observation for a specified time period. Id. at 63.

n136. This conclusion follows from the "T[su"] Rule" of statistics, which holds that, for a random process that follows the normal distribution, the standard deviation is proportional to the square root of time (or, equivalently, the number of observations). See id. at 31. To understand the T[su"] Rule, first note that (1) the variance of a time series of data (for example, of market returns) is equal to the sum of the expected squared deviations from the expected value (or return); and (2) the standard deviation is simply the square root of the variance. See Brealey & Myers, supra note 16, at 132-33. The T[su"] Rule then can be understood from the following: "Because variance is approximately proportional to the length of time interval over which a security or portfolio return is measured, standard deviation is proportional to the square root of the interval." Id. at 136 n.10. For example, if a market is a random process and follows a normal distribution, then the variance of weekly (five-day) returns should be five times the variance of daily returns - in other words, weekly variance equals five times daily variance. Because the standard deviation is the square root of the variance, daily returns can be scaled to weekly returns by multiplying the standard deviation of the daily return by the square root of five. Similarly, monthly return variances are often annualized by multiplying their standard deviations by the square root of 12. In the context of the hydrologist, range is functionally equivalent to standard deviation because, under the T[su"] Rule, range should increase with the square root of time (or, equivalently, the number of observations). See Peters, supra note 133, at 62-63; see also Schroeder, supra note 17, at 129-30 (describing R/S analysis in the context of "black noises").

n137. The square root of a number is equal to that number raised to the power of .50. Because the number of observations is the functional equivalent of time (T), random processes should be a function of T[su'.50'], or T[su"]. Hence, the T[su"] rule is so named.

n138. Hurst was working on the Nile River Dam project from 1907 through the late 1940s when he developed R/S analysis. See Peters, supra note 133, at 62-63 (summarizing H.E. Hurst, Long-Term Storage Capacity of Reservoirs, 116 Transactions Am. Soc'y Civ. Engineers 770 (1951)); see also Schroeder, supra note 17, at 129-30 (describing H in the context of river floods).

n139. Peters, supra note 133, at 63; Schroeder, supra note 17, at 130.

n140. Such a system is also called antipersistent or ergodic. Peters, supra note 133, at 64. Many studies using other statistical techniques have been conducted over the past several years identifying mean reversion in stock-price behavior. E.g., Werner F.M. DeBondt & Richard H. Thaler, Anomalies: A Mean-Reverting Walk Down Wall Street, J. Econ. Persp., Winter 1989, at 189; Langevoort, supra note 5, at 864 n.43; Poterba & Summers, supra note 75.

n141. Such a system is also called trend-reinforcing. Peters, supra note 133, at 65.

n142. Id. at 64.

n143. A number raised to the power of one equals that number; a number raised to the power of zero equals one; and a number raised to the power of negative one equals the reciprocal of that number. See Daintith & Nelson, supra note 29, at 123 (definition of exponent).

n144. Peters, supra note 133, at 67 ("If H = 0.6, there is, in essence, a 60 percent probability that, if the last move was positive, the next move will also be positive.").

n145. See Peters, supra note 133, at 181.

n146. In the terms of nonlinear dynamics, the average cycle length of a system is the length of time after which "memory of initial conditions is lost." Id. at 82, 100-01; see infra notes 194-98 and accompanying text.

n147. Peters, supra note 133, at 81-103 (reporting applications to stock, bond, and currency markets, as well as to raw macroeconomic indicators). This is an important achievement that distinguishes nonlinear testing from most tests of the ECMH, which usually are limited to common stocks traded on the New York Stock Exchange.

n148. Id. at 84.

n149. Id.

n150. Id. at 84-85.

n151. Id. at 85, 101 ("In statistical terms, [each fourth anniversary] is the decorrelation time of the series.").

n152. Peters discloses both of these possibilities. See id. at 74-75. A distribution having high peaks and fat tails is called leptokurtotic. Id. at 28.

n153. See id. at 75.

n154. Id. at 85.

n155. In connection with his study of the S&P Index, Peters also computed the H exponent for various individual common stocks and made several interesting observations, one of which is especially noteworthy. See id. at 86-90 (finding that a number of individual stock prices exhibited high H values, thus indicating persistence rather than randomness). First, note that H is a measure of randomness with respect to a security or a system and that the lower the H (above

.50), the more randomness in the security's or system's behavior; the higher the H (above .50), the less randomness, the more persistence, and the clearer the trends. Id. at 89. Thus, intuitively - and in accordance with modern portfolio theory, see supra notes 96-104 and accompanying text - the S&P Index's H exponent was higher than the H exponent for any individual stock tested. Peters, supra note 133, at 89-90. Put another way, low H exponents (above .50) denote higher risk because the subject exhibits more randomness and less predictable behavior than subjects with higher H exponents. See id. at 89. In Peters's study, however, the H exponents for the individual stocks were inversely related to the bs of those stocks. See id. at 87-89. This raises an important question for measuring risk: Is risk better measured in terms of the variability of a security versus a market portfolio (as in b, measured in terms of statistical variance) or in terms of the variability of the security itself over time (as in H, measured in terms of the jaggedness of a security's price behavior over time)? It is possible that the jaggedness H measures is, in the terms of modern portfolio theory described above, simply nonsystematic risk, which therefore can be diversified away. See supra notes 96-104 and accompanying text. It is also very likely, however, that H is measuring something that b cannot measure. Recall that the CAPM assumes rational expectations and linearity. See supra notes 105-15 and accompanying text. H does not make any such assumptions and therefore it gives recognition to the possibility that the rational-expectations assumption and the assumption of a linear process may not be true. Given the well-known deficiencies in specifying b, H may well provide a superior alternative measure of risk. Indeed, as this Article went to press, Peters published another book expanding on this theme. Edgar E. Peters, Fractal Market Analysis: Applying Chaos Theory to Investment and Economics (1994).

n156. Many of the tests are presented in their book, Brock et al., supra note 133, a rich, although difficult, discussion and analysis of a number of studies and techniques in nonlinear dynamics and chaos theory as applied to public capital markets. For the empirical results of their stock market studies, see id. at 82-129. In addition to the tests discussed in the text, the authors also conducted the following tests: the BDS statistic (named for Brock, W. Dechert, and Jose Scheinkman, in an unpublished paper. See id.); the Engle test for ARCH (acronymously named for Robert F. Engle's work, Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation, 50 Econometrica 987 (1982)); and the Tsay test for nonlinearity (named for R. Tsay, Nonlinearity Tests for Time Series, 73 Biometrica 461 (1986)). See Brock et al., supra note 133, at 41-86. Each of these tests involves many complex statistical techniques that are beyond the scope of this Article. More technical discussions of the tests discussed in the text may be found in William A. Brock, Distinguishing Random and Deterministic Systems: Abridged Version, 40 J. Econ. Theory 168 (1986); J.P. Eckmann & D. Ruelle, Ergodic Theory of Chaos and Strange Attractors, 57 Revs. Mod. Physics 617 (1985).

n157. See supra note 29 and accompanying text.

n158. The dimension is simply a measure of the number of variables or degrees of freedom of the system generating the data. See supra note 32.

n159. This is called creating a phase space of the time series, a subject discussed in greater detail infra text accompanying notes 200-05.

n160. See Brock et al., supra note 133, at 15, 42, 84; Peters, supra note 133, at 152-53; see also David Ruelle, Can Nonlinear Dynamics Help Economists?, in The Economy as an Evolving Complex System, supra note 32, at 195, 200 (illustrating this technique and citing its first publication, N.H. Packard et al., Geometry from a Time Series, 45 Physical Rev. Letters 712 (1980)).

n161. Brock et al., supra note 133, at 84. The distance is specified as a number of standard deviations in the data.

n162. More precisely, the logarithm of the correlation integrals and the logarithm of the specified distances are plotted against each other. Logarithms are used for statistical and mathematical convenience (e.g., multiplication and division of numbers can be conducted by addition and subtraction of their logarithms). The logarithm of a number (x) is the exponent indicating the power to which some other number (called the base) must be raised to produce the original number (x). For example, the logarithm of x with n as the base equals a in the formula n[su'a'] = x. See Daintith & Nelson, supra note 29, at 203.

n163. See Brock et al., supra note 133, at 88 ("Large dark areas on the recurrence plot indicate that many of the close points are occurring within a small segment of time. Light areas on the recurrence plot indicate that distant points are occurring within a small segment of time. This indicates violation of independent and identical distribution."). The term independent and identical distribution (IID) is a precise statistical term of art that roughly means randomness. See supra note 32. The authors continue:

We can infer the type of violation of IID by the patterns of light and dark areas in the recurrence plot. If (t,s) is darkened implies that (t+1,s+1) is likely to be darkened, the recurrence plots will have streaks parallel to the diagonal, which means that near neighbors can help to forecast future values. When time trends are present, the recurrence plot tends to fade out from the diagonal. During a period of reduced volatility, the recurrence plot contains darker squares symmetric about the diagonal. Finally, when the dynamics change abruptly between two periods, the recurrence plot displays an abrupt change of shading, making the recurrence plot a good indicator of structural shifts in economic and financial data.

Brock et al., supra note 133, at 88.

n164. Nonlinear dependence is greater during higher volatility periods, perhaps because of changes in the nature of beliefs - when beliefs are more heterogeneous there is more volatility, and when beliefs are more homogeneous there is less volatility. A test is needed to determine whether nonlinear dependence occurs independently of volatility levels. See Brock et al., supra note 133, at 86. The authors explain:

In financial time series much of the nonlinear dependence occurs in volatility. This is generally believed to arise from adjustments to new information. During turbulent times, the market is reacting to the inflow of new information, so beliefs are relatively heterogeneous and volatility is high. During quiet times, beliefs are more homogeneous and much of the volatility comes from liquidity trading. While volatility is correlated, the actual price changes remain uncorrelated.

Id.

n165. See id. at 86-87.

n166. The CRSP Index is the index maintained by the Center for Research of Securities Prices at the University of Chicago and is often used in empirical work concerning public capital market behavior.

n167. For example, they checked to see whether any of the anomalies - such as the week-of-the-month effect or the January effect, see Lindgren, supra note 9, at 16-18 - were present, creating linear dependence in the data. See Brock et al., supra note 133, at 95.

n168. Brock et al., supra note 133, at 96-97.

n169. Id. at 97.

n170. Id.

n171. Id. at 98.

n172. Id.

n173. Id. at 99.

n174. Id.

n175. Id.

n176. Id.

n177. See id. at 104-07.

n178. Id. at 29, 106-07.

n179. See id. at 106-07; see also supra notes 78-93, 164 and accompanying text.

n180. See Brock et al., supra note 133, at 29, 102, 105, 107.

n181. Id. at 29. They point out as well that the presence of nonlinear dependence also is consistent with "other (stochastic) dependent processes that are not chaotic." Id.

n182. E.g., Hinich & Patterson, supra note 132, at 76 (bispectrum analysis showed nonlinearity in stock returns, indicating dependence in the data and finding that "the mechanism responsible for nonlinearity in the process generating stock returns is similar across stocks"); David A. Hsieh, Chaos and Nonlinear Dynamics: Application to Financial Markets, 46 J. Fin. 1839, 1873-74 (1991) (finding "strong evidence to reject the hypothesis that stock returns are IID" but attributing this phenonmenon to "predictable variance changes" rather than chaotic dynamics); James B. Ramsey, Economic and Financial Data as Nonlinear Processes, in The Stock Market: Bubbles, Volatility, and Chaos 81 (Gerald P. Dwyer, Jr. & R.W. Hafer eds., 1990) [hereinafter Bubbles]; Scheinkman & LeBaron, supra note 34 (correlation dimension tests "point towards the presence of nonlinearities" and show the "inadequacy" of the random walk model, id. at 334, because they suggest an important role of nonlinear functions in explaining public capital market behavior, see id. at 312).

n183. See Barry B. Burr, Chaos: New Market Theory Emerges, Pensions & Investment Age, July 10, 1989, at 3, 3 (quoting Professor LeBaron as saying "It's very clear there is nonlinear dependence in financial data .... but it's difficult to tell if chaos is possible").

n184. See David Berreby, Chaos Hits Wall Street, Discover, Mar. 1993, at 76, 84 (discussing Prediction Company, a private company operated by a group of prominent physicists ("chaologists") under contract with O'Connor Associates, an affiliate of Swiss Bank Corporation and one of the most prominent firms engaged in options and futures trading in the world); Matt Ridley, Frontiers of Finance: On the Edge, The Economist, Oct. 9, 1993, at 3, 3 (claiming that "up to a dozen firms are now managing more than $100M each on the basis of advice generated by computers" invoking nonlinear dynamics and chaos theory); Gary Weiss, Chaos Hits Wall Street - The Theory, That Is, Bus. Wk., Nov. 2, 1992, at 138, 138-40 (identifying a "small but growing" body of "chaologists" - money managers using the mathematical techniques of nonlinear dynamics to model public capital markets - and reporting that the New York Society of Securities Analysts sponsored a day-long conference on chaos and markets in October 1992). The New York Society of Securities Analysts sponsored another conference on chaos and capital markets in 1993.

n185. See Weiss, supra note 184, at 139-40.

n186. The value of the ECMH could be equated to that of a theory explaining why when you throw a ball into the air it does not fall back to earth. This would not necessarily mean, however, that the ECMH is not valuable to economists or to investors in explaining portions of market processes - the public capital markets alternately may obey a stochastic and a deterministic process, involving both random and nonrandom components. See Tonis Vaga, The Coherent

Market Hypothesis, Fin. Analysts J., Nov.-Dec. 1990, at 36 (developing a statistical model along such lines). But this partial validity must not be misunderstood to suggest that markets are "relatively efficient," "reasonably efficient," "sufficiently efficient," or "more efficient than not" at any moment in time, as many devotees of the ECMH have been forced to argue since the public capital market crash of 1987 shook confidence in that theory. E.g., Brealey & Myers, supra note 16, at 297-300; Fischel, supra note 82, at 913-15. Rather, markets may be relatively efficient only on some days but not on all days. Accordingly, although a financial-economic theory that explains only a portion of a system may be useful to economists, it would be difficult to defend transplanting such a partial theory to the domain of public policy articulated by lawyers. Cf. Lawrence E. Mitchell, The Cult of Efficiency, *71 Tex. L. Rev. 217, 224 n.31 (1992)* (reviewing Frank H. Easterbrook & Daniel R. Fischel, The Economic Structure of Corporate Law (1991)) (noting the dangers of building legal policy on economic theories that might not work).

n187. See infra part III.A.

n188. James Gleick, Chaos: Making a New Science (1987).

n189. The Economy as an Evolving Complex System, supra note 32. The Santa Fe Institute was founded in 1984 to research and to study complex systems, drawing on advanced techniques from many disciplines, including physics, mathematics, economics, and others. Id. at overleaf.

n190. E.g., Bubbles, supra note 182; H.W. Lorenz, Nonlinear Dynamical Economics and Chaotic Motion (2d ed. 1993); William J. Baumol & Jess Benhabib, Chaos: Significance, Mechanism, and Economic Applications, 3 J. Econ. Persp. 77 (1989); Jean-Michel Grandmont & Pierre Malgrange, Nonlinear Economic Dynamics: Introduction, 40 J. Econ. Theory 3 (1986) (introducing an entire issue devoted to economic models that appeared random but in fact were deterministic); see also Simmonds et al., supra note 1, at 144 n.71 (citing many other sources).

n191. See sources cited supra note 1.

n192. Cf. supra note 32.

n193. See A Concise Dictionary of Physics 38 (Oxford Univ. 1990) (defining chaos as "unpredictable and seemingly random behavior occurring in a system that should be governed by determinstic laws").

n194. For a fine and easily accessible essay concerning the three-body problem and chaos theory written for college physics students, see J.C. Sprott, Essay: Chaos, in Raymond A. Serway, Physics 423 (3d ed. 1990).

n195. Daintith & Nelson, supra note 29, at 319.

n196. Peters, supra note 133, at 135.

n197. Id. at 133 (quoting Poincare).

n198. Brock et al., supra note 133, at 9. The classic example of sensitivity to initial conditions is the butterfly effect in meteorology: "The dynamical equations governing the weather are so sensitive to the initial data that whether or not a butterfly flaps its wings in one part of the world may make the difference between a tornado occurring or not occurring in another part of the world." A Concise Dictionary of Physics, supra note 193, at 38-39. The term butterfly effect was taken from the paper by Edward Lorenz in which sensitive dependence on initial conditions was first propounded. See Gleick, supra note 188, at 322 n.20.

n199. I am indebted to Professor Alan Wolf for this analogy and its exposition, elaborated in Alan Wolf, Chaos in the Stadium, Algorithm, Apr. 1992, at 17. In the example given, the billiard table is assumed to be semicircular rather than rectangular. A rectangular billiards table is not chaotic because as the ball travels the geometry of the table, initial measurement error amplifies linearly. In contrast, a semicircular table (or a similar area, such as a hockey rink), is chaotic in that as the ball (or puck) travels the geometry of the area, initial measurement error amplifies exponentially.

n200. This discussion ultimately is informed by many sources, including Gleick, supra note 188, at 132-53; Peters, supra note 133, at 133-40; Berreby, supra note 184, at 80-81. The illustrations, set forth infra pp. 585-87, were prepared for this Article by Professor Alan Wolf, to whom I am very grateful. Additional visual illustrations may be found in Gleick, supra note 188, at 136-37; Berreby, supra note 184, at 80. For a lucid text that balances accessability and technical sophistication, see Baker & Gollub, supra note 128.

n201. See fig. A.

n202. See fig. B.

n203. See fig. C.

n204. See fig. D.

n205. Compare figs. E and F. Figure E depicts the time series of a simulated weather system, suggesting behavior that is completely random. Figure F depicts a phase portrait of the same system, revealing a strange attractor.

n206. Peters, supra note 133, at 147.

n207. See id.

n208. See id. (LEs "measure how quickly nearby orbits diverge in phase space.").

n209. Id.

n210. Id. Consider the following:

Imagine an undamped pendulum placed on a table and swinging in regular motion. Someone bumps the table and causes the pendulum to lose its rhythm. However, if there is no other disturbance, the pendulum will settle back to a steady rhythm with a new amplitude. In phase space, the pendulum's orbit is characterized by a closed circle, or limit cycle. If we were to plot the action when the table is bumped, we would see some orbits swing wildly away from the limit cycle, before settling into a new limit cycle. The negative Lyapunov exponent measures the number of orbits, or the amount of time, it takes for the phase plot to return to its attractor, which in this case is a limit cycle.

Id.

n211. See id.

n212. See Gleick, supra note 188, at 253; Peters, supra note 133, at 147.

n213. Peters, supra, note 133, at 148. Information theory is a common term used to describe the mathematical theory of communication. Information theory investigates, among other things, how to measure and transmit information through computers and other digitized communication devices in which data are stored in binary format. See id.

n214. Id.

n215. See id.

n216. Id. at 148-49.

n217. Id.

n218. Id. at 149. To calculate a full spectrum of LEs for a system (one for each dimension) would require knowledge about the motion of the system; in terms of physics, we would need to know the equation of motion of the system. This is not currently possible for capital markets. A method has been developed, however, to enable calculation of the largest LE of a system, without knowing the equation of motion of the system. Alan Wolf et al., Determining Lyapunov Exponents from a Time Series, 16D Physica 285 (1985). If the largest LE is positive, then the system exhibits sensitive dependence on initial conditions and contains a strange attractor. Calculating the largest LE is a multi-step statistical operation that includes several estimates and variable selections. As in all statistical analysis, the first questions are what interval to choose and how long the series should be. In chaos analysis, however, standard statistical rules do not necessarily apply. Thus, in standard statistics, more data is generally regarded as better; in chaos analysis, it is more important to study longer time periods than to study more data. See Peters, supra note 133, at 173-74; see also H.S. Greenside et al., Impracticality of a Box-Counting Algorithm for Calculating the Dimensionality of Strange Attractors, 25 Physical Rev. A 3453 (1982) (original article demonstrating the problem of procuring adequate data for high-dimensional chaos analysis); Alan Wolf & Tom Bessoir, Diagnosing Chaos in the Space Circle, 50D Physica 239, 250-55 (1991) (identifying additional problems with measuring chaotic systems). Once a data interval and length are chosen, the time series is reconstructed into phase space. See id. at 252; see also supra notes 201-04 and accompanying text. It also may be necessary in chaos analysis to examine prices directly rather than price changes. Using price changes is customary in linear analysis simply because, as is well known, there are strong trends in prices themselves, largely as the result of inflation. To examine prices directly therefore first requires "detrending" the data to remove the effects of inflation. See Peters, supra note 133, at 163-68. The largest LE then can be calculated by taking the following steps, all of which are described by Peters, see id. at 171-78. First, two points are chosen, requiring the choice of the distance between them. Second, the distance between those points at some subsequent period is measured, requiring the choice of a lag time, called the evolution period. Because the largest LE is being measured solely to measure divergence and not contraction in phase space, if the two points initially selected were to diverge too far apart by the end of the evolution period, they would begin to converge, and only this convergence would be measured. Therefore, if the points diverge too much, a replacement point must be chosen. Thus, how much divergence in the orbits of those points should be considered too much also must be determined. The dimension or number of degrees of freedom also must be determined. Finally, the calculation itself must converge to a stable value of LE or else either the system is not nonlinear or the foregoing parameters were misspecified. Although choices of these variables are constrained by statistical theory, each requires substantial judgment.

n219. There appears to be a typographical error in Peters' book: in one place it specifies 1989, see Peters, supra note 133, at 166; in another, it specifies 1990, see id. at 177.

n220. Peters chose monthly data because it balances the need for minimizing calculation time with the need for adequate replacement points. Id. at 176.

n221. Id. at 178. This calculation was based on a dimension of four, a time lag of twelve months, and an evolution time of six months. Id. at 177. Peters determined that the dimension (m) had to be at least three - "the embedding dimension should be higher than the fractal dimension, because a rough surface often looks smoother when placed in a higher dimension" - and he found that the fractal dimension of the S&P Index was 2.33. Id.; see infra text accompanying notes 225-34. He then calculated the LE using dimensions of three and greater and found that the LE converged to a stable level when the dimension was four. See id. at 177-78. Given an average cycle length of four years (48 months), see supra note 151, a time lag of 16 months would be necessary for $m = 3$ ($48/3 = 16$), and a time lag of 12 months would be necessary for $m = 4$ ($48/4 = 12$).

n222. See id. at 178.

n223. See supra note 151 (showing an average cycle length of approximately four years using R/S Analysis). The results match "substantially" because the 42-month period shown by the LE calculation "is roughly equal to the 1,000-day trading cycle obtained using R/S Analysis." Peters, supra note 133, at 178.

n224. See J.-P. Eckmann et al., Lyapunov Exponents for Stock Returns, in The Economy as an Evolving Complex System, supra note 32, at 301, 301-04 (reporting discovery of positive LEs in stock market data).

n225. For a more elaborate overview of fractal geometry, see Peters, supra note 133, at 45-60 (containing amplification and fascinating pictorial illustration of fractal shapes, including the Sierpinski Triangle and the Koch Snowflake). More colorful pictorials are included in Gleick, supra note 188, at 114-15, and H.O. Peitgen & P.H. Richter, The Beauty of Fractals: Images of Complex Dynamical Systems (1986). See also Schroeder, supra note 17, at 177-236 (more detailed discussion of fractals as well as colorful pictorials).

n226. See Peters, supra note 133, at 45.

n227. Benoit B. Mandelbrot, The Fractal Geometry of Nature (updated ver. 1982).

n228. Id. at 1.

n229. In mathematical terms, it is not completely differentiable across its entire surface. See Peters, supra note 133, at 55.

n230. See id.

n231. See id. at 56.

n232. See id. at 57.

n233. Id. These properties distinguishing random from nonrandom time series may be conceptualized in a different way. For example, our conception of a crumpled piece of paper as a three-dimensional object can be regarded as embedding a fractal in a dimension greater than itself. That greater dimension is called the embedding dimension. See id. at 56. Fractals retain their fractal dimension when placed in an embedding dimension; random distributions do not. Thus, unlike nonrandom distributions, random distributions fill their space the way gas fills a volume - the gas spreads out because there is nothing to bind the molecules together. See id. at 56-57. This is, of course, the defining characteristic of Brownian motion.

n234. Id. at 170 tbl. 13.1.

n235. These tests were conducted along the lines discussed supra notes 152-55 and accompanying text.

n236. Id. at 184.

n237. See supra notes 57-77 and accompanying text.

n238. See supra notes 78-93 and accompanying text.

n239. See, e.g., supra notes 79 and 81 and accompanying text.

n240. See supra note 155 and accompanying text (R/S Analysis and H-exponent tests); supra notes 213-24 and accompanying text.

n241. The perspective developed in this Article implies that every legal policy and normative claim alleged to be supported by the ECMH, even in the diluted form left by noise theory, warrants review and positioning in the more complex context of nonlinear dynamics and chaos theory. But cf. infra note 311 (cautioning policymakers against undue reliance on ECMH and noise theory as well as on nonlinear dynamics and chaos theory).

n242. The Dow Jones Industrial Average (the Dow) was in the 2600 range in early October 1987; on October 19, 1987, the Dow fell 508 points, to 1738.74. The crash was not limited to the 30 common stocks comprising the Dow but was worldwide. For example, the New York Stock Exchange, the London Stock Exchange, and the Tokyo Stock Exchange all crashed. See Report of the Presidential Task Force on Market Mechanisms 30 (1988) [hereinafter Brady Report].

n243. Among the bits of information are the following: On September 4, 1987, the Federal Reserve Board raised the discount rate; on October 13, 1987, the House Ways and Means Committee voted to approve income tax legislation that would disallow interest deductions paid on indebtedness used to finance business acquisitions; on October 18, 1987, Treasury Secretary Baker publicly announced an intention to reduce the value of the dollar; and market prices were already high by historical standards. See S. Rep. No. 300, 101st Cong., 2d Sess. 30-33 (1990); Brady Report, supra note 242, at 15-44. The Brady Report also attributed the market crash to such institutional factors as program trading, portfolio insurance, and derivative securities. Brady Report, supra note 242, at 29-42. In the light of the international nature of the crash, however, virtually no one has given much credence to these suggestions. E.g., David D. Haddock, The Swiftness of Divine Retribution and Its Tendency to Mistake Its Target: An Analysis of the Brady Report, in Bubbles, supra note 182, at 179, 180; David D. Haddock, An Economic Analysis of the Brady Report: Public Interest, Special Interest, or Rent Extraction?, *74 Cornell L. Rev. 841 (1989).*

n244. E.g., Brealey & Myers, supra note 16, at 299 ("There was no obvious, new fundamental information to justify such a sharp decline in share values."); Shleifer & Summers, supra note 87, at 29 (no evidence of macroeconomic forces such as increased risk or decreased dividend rates).

n245. See, e.g., Simmonds et al., supra note 1, at 148-50 (attributing the October 1989 market panic to collapse of an attempted United Airlines takeover and the November 1991 market drop to speculation about cuts in credit-card interest rates).

n246. See supra notes 87-91 and accompanying text.

n247. See Fischel, supra note 82, at 915 ("To date, no convincing explanations of this dramatic decline exist."). Many have suggested that psychological factors play a substantial role in market crashes. See Shiller, supra note 87, at 3-4 (describing a popular model consisting of comparing the sequence of price movements surrounding the 1929 market crash with those that might unfold sometime in the future); Robert J. Shiller, Speculative Prices and Popular Models, J. Econ. Persp., Spring 1990, at 55; Shleifer & Summers, supra note 87, at 28 (crash probably attributable to "positive feedback strategies" or "trend chasers" - investors who buy stocks as and after they rise and sell stocks as and after they fall); see also SEC Div. Market Reg., The October 1987 Market Break, at xiii (1988) (comprehensive analysis of the 1987 market crash by the staff of the Division of Market Regulation concluding that "no single factor - economic, structural or psychological - was responsible for the size and breadth of the October 1987 market break").

n248. See Brock et al., supra note 133, at 14 ("It is not patently ridiculous to imagine that such events as the stock market crash of October 19, 1987, might have been a display of a route to chaos.").

n249. See id. (explaining that it is possible to imagine that ""economic' time speeds up during [market breaks] whereas data are collected in chronological time"); Richard B. Olsen et al., Going Back to Basics: Rethinking Market Efficiency 2 (1992) (monograph) (copy on file with the author).

n250. See Olsen et al., supra note 249, at 2.

n251. Genesis 41:25-31; see Peters, supra note 133, at 108, 233.

n252. See supra notes 134-65 and accompanying text.

n253. Genesis 7:17-24; see Peters, supra note 133, at 108, 234.

n254. See Brock et al., supra note 133, at 14.

n255. Professors Gennotte and Leland have developed a rational-expectations model that builds on this discontinuity thesis by defining the occurrence of a market crash as a result of "discontinuity in the relationship between the underlying environment and stock prices: an infinitesimal shift in information (or other small shock) can lead to a major change in stock market level." Gerard Gennotte & Hayne Leland, Market Liquidity, Hedging, and Crashes, *80 Am. Econ. Rev. 999, 1000 (1990).* Gennotte and Leland point out that "such discontinuities are commonly observed in physical systems and have been the recent subject of study by mathematicians examining "catostrophe theory."' Id. at n.1. Catastrophe theory is a branch of chaos theory.

n256. See Olsen et al., supra note 249, at 2.

n257. Id. at 3.

n258. See id. Incremental information changes in a perfect market would be expected to produce proportionate price changes. In existing markets, informational changes produce disproportionate changes. In terms of chaotic dynamics, these disproportionate changes may be seen as a result of initial measurement error that (as in the billiard table example and the butterfly effect generally, see supra notes 194-99 and accompanying text), leads to exponentially greater price changes over time.

n259. The subject of market structure, including transparency, fragmentation, and immediacy, is well-discussed in several pieces in Modernizing United States Securities Regulations: Economic and Legal Perspectives (Kenneth Lehn & Robert W. Kamphuis, Jr. eds., 1992) [hereinafter Modernizing Regulations]. See Corinne M. Bronfman, If It Ain't Broke, Don't Regulate It, in Modernizing Regulations, at 407; J. Harold Mulherin, Market Transparency: Pros, Cons, and Property Rights, in Modernizing Regulations, at 375; Robert A. Schwartz, Competition and Efficiency, in Modernizing Regulations, at 383; Hans R. Stoll, Organization of the Stock Market: Competition or Fragmentation?, in Modernizing Regulations, at 399.

n260. See Schwartz, supra note 259, at 388-89.

n261. See id. at 383-84.

n262. Id. at 387.

n263. Id.

n264. Id. at 384.

n265. See Bronfman, supra note 259, at 409; Schwartz, supra note 259, at 386.

n266. Price discovery in such continuous markets is obscured because (1) in quote-driven markets like NASDAQ, public participants do not place priced orders; instead, dealers post quotes at which the public may trade; and (2) even in order-driven markets like the NYSE, although some public participants use limit orders to establish the price at which their own transactions will be executed, those limit orders are then used to establish the price at which other public participants may trade. See Schwartz, supra note 259, at 385-86.

n267. See id. at 384; Stoll, supra note 259, at 402-03.

n268. Schwartz, supra note 259, at 389-90.

n269. Professor Schwartz has suggested that these market imperfections could be addressed by alternative market structures such as call markets. See id. at 385-92. The SEC's Market 2000 study also addresses some of these subjects. See SEC Div. of Mkt. Regulation, Market 2000: An Examination of Current Equity Market Developments [1994 Transfer Binder] *Fed. Sec. L. Rep. (CCH) 85,311* (Jan. 27, 1994).

n270. For example, noise theory ultimately is a critique of the ECMH on its own terms, because it implies simply a continuum of relative efficiency under which markets with high levels of noise are relatively less fundamentally efficient and relatively more informationally efficient.

n271. In addition to the examples discussed in the text, as a matter of securities regulatory policy, the ECMH has been invoked to debate the proper definition of materiality. See Dennis, supra note 7, at 374-81. The ECMH also has been used to evaluate the scope of mandatory disclosure rules. See Easterbrook & Fischel, Mandatory Disclosure, supra note 7, at 694 (suggesting that because of ECMH, mandatory disclosure makes many investors marginally worse off); see also Bebchuk, supra note 7 (assessing tender-offer regulation in the light of ECMH); Easterbrook & Fischel, Corporate Control Transactions, supra note 7 (same); Fischel, supra note 7 (same). But see Lawrence A. Cunningham, Firm Specific Information and the Federal Securities Laws: A Doctrinal, Etymological and Theoretical Critique, 68 Tul. L. Rev. (forthcoming 1994) (assessing Easterbrook & Fischel's Mandatory Disclosure article). In corporate law, the ECMH was used to defend the stock-market exception to the appraisal remedy. See Del. Code Ann. tit. 8, 262(b) (1991) (appraisal remedy for dissenting shareholders is not available in mergers for stock or sales of assets for stock where the stock is listed on a national securities exchange); Richard M. Buxbaum, The Dissenter's Appraisal Remedy, *23 UCLA L. Rev. 1229, 1247-48 (1976)* (arguing against the public-market exception to appraisal rights); Alfred F. Conard, Changes in the Model Business Corporation Act Affecting Dissenters' Rights, *32 Bus. Law. 1855, 1862-63 (1977)* (same); Joel Seligman, Reappraising the Appraisal Remedy, *52 Geo. Wash. L. Rev. 829, 837-40 (1984)* (arguing the merits of the appraisal remedy in the light of the ECMH). The hypothesis also figured prominently in debate concerning the policy basis for shareholder derivative lawsuits. See John C. Coffee Jr., Litigation and Corporate Governance: An Essay on Steering Between Scylla and Charybdis, *52 Geo. Wash. L. Rev. 789, 807-08 (1984)* (arguing that the ability to diversify has repercussions for the purposes of derivative suits); James D. Cox, Compensation, Deterrence, and the Market as Boundaries for Derivative Suit Procedures, *52 Geo. Wash. L. Rev. 745, 748-49 (1984)* (same).

n272. The adoption of circuit breakers was premised in part on the informational/fundamental distinction of noise theory. See Ayres, supra note 6, at 981 n.143; supra notes 78-93 and accompanying text.

n273. E.g., Brady Report, supra note 242, at 66 (Circuit breakers include "price limits, position limits, volume limits, trading halts reflecting order imbalances, trading halts in derivatives associated with conditions in the primary marketplaces, and the like.").

n274. E.g., N.Y.S.E. Listed Company Manual, Rule 80B (if the Dow falls 250 points during a single trading day, trading in all listed stocks is halted for one hour).

n275. See, e.g., S. Rep. No. 300, supra note 243, at 31-34; Brady Report, supra note 242, at 57.

n276. See Bruce Greenwald & Jeremy Stein, The Task Force Report: The Reasoning Behind the Recommendations, J. Econ. Persp., Summer 1988, at 3, 7-8.

n277. Professor Ayres has noted this problem:

The early declines in futures contract prices [trading on the Chicago Mercantile Exchange on a day when circuit breakers were triggered] increased the probability that the circuit breaker would be activated. This increased probability had a destabilizing effect as the market discounted the security's value because of the impending possibility of illiquidity. It follows that the prospect of illiquidity could have caused the circuit breaker to have a magnetic effect, pulling down security prices that came too close: "when prices approach a circuit-breaker level, the drop accelerates in a rush to sell before the mechanism is triggered."

Ayres, supra note 6, at 981 (quoting Eben Shapiro, Circuit Breakers: Maybe They Work, Maybe They Don't, N.Y. Times, July 29, 1990, at F7); see also Lewis D. Solomon & Howard B. Dicker, The Crash of 1987: A Legal and Public Policy Analysis, *57 Fordham L. Rev. 191, 237 (1988)* (citing this "gravity effect," as well as problems of intermarket coordination of circuit breakers and general concerns with artificial constraints on the market, as the problems with price-level halts).

n278. See Shapiro, supra note 277, at F7. This rationale was relied on expressly in the Brady Report's recommendations supporting adoption of circuit breakers. Brady Report, supra note 242, at 66.

n279. See supra notes 251-54 and accompanying text.

n280. See Allen D. Boyer, Activist Shareholders, Corporate Directors, and Institutional Investment: Some Lessons from the Robber Barons, *50 Wash. & Lee L. Rev. 977, 990-91 (1993)* (describing the phenomenon of hyperefficiency, its causes, and its effects).

n281. Although bull markets do not generate the immediate concern that bear markets generate, a bull market feeding on excess liquidity can produce a false perception of sustainability that itself threatens a later crash. E.g., S. Rep. No. 300, supra note 243, at 30; see also Brady Report, supra note 242, at 9-14 (graphing the bull nature of U.S. and international markets prior to the 1987 crash). Further, as Allen Boyer has pointed out, hyperefficient markets in which booms feed on themselves also have implications for the market for corporate control:

In the hyperefficient market, a boom may feed on itself. When the securities market has grown accustomed to takeovers, buyouts, and restructurings, and has come to believe that these transactions of themselves create value, all public corporations may be viewed as potential takeover candidates .... if the market feels that takeovers and restructurings offer a short-cut to maximizing profits.

Boyer, supra note 280, at 991.

n282. See Greenwald & Stein, supra note 276, at 7-8 (explaining the difficulty of identifying the causes of the 1987 crash and making helpful policy recommendations).

n283. See supra notes 251-54 and accompanying text.

n284. Such models have been developed and are employed by Olsen & Associates, a Swiss trading firm, and its affiliated research group, The Institute for Applied Economics, in Zurich (collectively, Olsen). See Olsen et al., supra note 249. Olsen's measure of intrinsic time operates by applying statistical scaling laws of the kind described supra note 136, and holds that there is a close correlation between a specified time interval and specified amounts of price changes. Their model tracks this relationship and measures the departure from the expected relationship, indicating when intrinsic time speeds substantially ahead of chronological time. See id.

n285. Olsen et al., supra note 249.

n286. As a practical matter, the SIT circuit breaker is far less likely to be triggered in a bull market than in a bear market, however, because SIT accelerates relatively more slowly in bull markets than in bear markets. Cf. Kraakman, supra note 9, at 930 ("At the outset of bull markets, discounts have tended to increase, as funds' portfolio values have risen faster than their share prices; at the outset of bear markets, by contrast, discounts have tended to decrease, as portfolio values have fallen faster than share prices."). The SIT circuit breaker also lets markets correct themselves if the drop is not driven by a structural inability to process information as rapidly as intrinsic time is passing. In other words, under the price-level circuit breaker, prices can rise without limit and out of all proportion to information changes but they cannot fall too steeply even in proportion to information changes. The SIT circuit breaker cuts off trading in both disproportionate bull and disproportionate bear markets, but only based on the speed of intrinsic time and not in circumstances that permit the possibility that information changes justify the trend.

n287. The foregoing suggestions also should not be understood to imply that the Senate, the SEC, the authors of the Brady Report, or others involved in investigating the 1987 market crash did not meet the many challenges they faced in attempting to understand that crash. Rather, their indefatigable efforts in facing those challenges simply underscore how very complex the questions are.

n288. Langevoort, supra note 5, at 881. Professor Langevoort's overall analysis centered on shelf registration, integrated disclosure, and the fraud-on-the-market theory, although the irony he identified operates more broadly and would include mandatory disclosure rules. Cf. James D. Cox et al., Securities Regulation: Cases and Materials 686 (1991) ("One may simultaneously accept the noisiness of prices and still believe that formalized disclosure (e.g., delivery of a full prospectus) to the broad community of investors is not cost-justified with respect to larger companies about whom a large body of information is always accessible to investors.").

n289. See, e.g., Easterbrook & Fischel, supra note 186, at 276-334; Easterbrook & Fischel, Mandatory Disclosure, supra note 7, at 695-707; Jonathan R. Macey, Administrative Agency Obsolescence and Interest Group Formation: A Case Study of the SEC at Sixty, *15 Cardozo L. Rev. 909, 927-37 (1994).* Professor Macey has recently invoked the ECMH, CAPM, MPT, and noise theory to argue not only that mandatory disclosure rules should be abolished, but also that the SEC itself has become obsolete and therefore should be abolished. Id.

n290. Langevoort, supra note 5, at 881.

n291. Event-study methodology recognizes this complexity but then minimizes it. For example, in establishing the opening and closing dates of an event window to measure the effect of a discrete bit of information, there is a tradeoff between choosing a window long enough to assure including all dates on which the bit may have been recieved by traders and choosing one short enough to avoid infecting the study with other bits of information, which is what event-study methodology calls a "confounding event." See Macey et al., supra note 6, at 1030. Another problem with event-study methodology is its assumption that choosing a window-closing date is "straightforward[] because the market absorbs and processes information rapidly," id. at 1031, an assumption belied by the presence of nonlinear structure and chaotic behavior in public capital markets. Despite these limitations on event-study methodology, it is often used to analyze legal issues. E.g., Roberta Romano, The Genius of American Corporate Law (1993); Roberta Romano, The State Competition Debate in Corporate Law, *8 Cardozo L. Rev. 709 (1987).*

n292. Gleick, supra note 188, at 24.

n293. See SEC Div. of Mkt. Regulation, supra note 269. This discussion deliberately has isolated the effect of capital market theory on the debate concerning mandatory disclosure rules, without addressing the broader question of their proper goals or other mechanisms than mandatory disclosure rules that might address those goals.

n294. E.g., William J. Carney, The ALI's Corporate Governance Project: The Death of Property Rights?, *61 Geo. Wash. L. Rev. 898, 908-12 & n.68, 950-51 & n.307 (1993)* (arguing that mandatory fiduciary-duty rules do not survive a cost-benefit analysis in the face of the undisputed reality of ECMH). This position originated in the Jensen-Meckling conception of the corporation. Michael C. Jensen & William H. Meckling, Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure, 3 J. Fin. Econ. 305 (1976). Contra Victor Brudney, Corporate Governance, Agency Costs, and the Rhetoric of Contract, *85 Colum. L. Rev. 1403, 1403 (1985)* ("The "markets' for ... securities do not effectively implement investor constraints on management.").

n295. See Carney, supra note 294.

n296. See supra note 10 and accompanying text.

n297. Macey & Miller, supra note 83, at 1013.

n298. Id.

n299. Managers undoubtedly attempt to influence their company's stock price all the time in ways ranging from the mundane (casting unfavorable announcements in the best light subject to anti-fraud constraints) to extraordinary distributions to shareholders from the sale of debt or assets. Those attempts, however, are not motivated primarily by a desire to produce stock prices equivalent to underlying asset values. Even if they were, the effect of the undertaking is to address the so-called misinvestment hypothesis, one of the most common reasons cited for the disparity between stock prices and asset values in the first place. The misinvestment hypothesis claims that investors rationally doubt management's ability to deploy discretionary funds (free cash flow) in an optimal manner and discount share prices from fundamental values accordingly. Because debt-financed distributions address the misinvestment hypothesis only by reducing managerial discretion over future investments (reducing free cash flow), such distributions do not eliminate the fundamental constraint on managers' ability to set share prices equal to fundamental values. See Kraakman, supra note 9, at 914-15.

n300. Cf. Henry T.C. Hu, Risk, Time, and Fiduciary Principles in Corporate Investment, *38 UCLA L. Rev. 277, 282 (1990)* (fiduciary duties of corporate managers should require maximization of fundamental asset values, and managers should "blissfully disregard[] evidence that stock market pricing of shares is, to a disturbing extent, ill-informed and irrational"); Charles M. Yablon, Overcompensating: The Corporate Lawyer and Executive Pay, *92 Colum. L. Rev. 1867, 1886 n.52 (1992)* (reviewing Graef Crystal, In Search of Excess (1991)) ("Some observers, however, consider stock price to be a poor indicator of managerial performance.").

n301. The argument Professors Macey and Miller make seems to assume that the purpose of fiduciary duties is to maximize shareholder welfare defined strictly in terms of the prevailing price for a corporation's publicly traded stock. See Macey & Miller, supra note 83, at 1013. This way of understanding fiduciary duties seems unduly narrow, however, because fiduciary duties require corporate managers to act in the best interests of the corporation to promote its welfare. That welfare is usually not defined narrowly, as by reference to the corporation's stock price. Rather, it is defined broadly by reference to the general business and financial condition of the corporation. E.g., *Remillard Brick Co. v. Remillard-Dandini Co., 241 P.2d 66, 74 (Cal. Dist. Ct. App. 1952); Smith v. Van Gorkom, 488 A.2d 858, 872 (Del. 1985); Weinberger v. UOP, Inc., 457 A.2d 701, 710 (Del. 1983); Shlensky v. Wrigley, 237 N.E.2d 776, 779-80 (Ill. App. Ct. 1968); Kamin v. American Express Co., 383 N.Y.S.2d 807, 812 (Sup. Ct. 1976); Litwin v. Allen, 25 N.Y.S.2d 667, 678 (Sup. Ct. 1940).*

n302. This discussion, like the discussion of mandatory disclosure rules, deliberately has isolated the effects of capital market theory on mandatory fiduciary obligations, without addressing broader questions such as the appropriate beneficiaries of such duties and whether mechanisms other than mandatory duties might address its goals. For an excellent and thorough discussion of fiduciary obligation, see Lawrence E. Mitchell, Fairness and Trust in Corporate Law, *43 Duke L.J. 425 (1993).*

n303. Mark Roe, The Modern Corporation and Private Pensions, at 35-36 (1993) (unpublished manuscript, on file with the author). But see Ronald J. Gilson & Reinier Kraakman, Reinventing the Outside Director: An Agenda for Institutional Investors, *43 Stan. L. Rev. 863 (1991)* (arguing that institutional investors should become involved in corporate governance).

n304. See supra notes 96-104 and accompanying text.

n305. See supra notes 106-23 and accompanying text.

n306. Roe, supra note 303, at 36.

n307. See id. Professor Roe states:

The dominant nonacademic standard is that trading by sophisticated analysis improves returns; the dominant academic standard is to construct a diversified portfolio and sit with it. Both fit managers' goals. We do not yet have an equally prominent theory that extols the virtue of coupling large block investing with boardroom presence.

Id.

n308. See Louis Lowenstein, What's Wrong with Wall Street 7 (1988).

n309. See supra note 155. Note that nonlinear dynamics and chaos theory do not refute the intuitive insight that portfolio diversification is sensible.

n310. E.g., Ian Ayres & Peter Cramton, Relational Investing and Agency Theory, *15 Cardozo*

L. Rev. 1033 (1994) (arguing that "relational investing can reduce agency costs," id. at 1035, and perform a number of other corporate governance functions); Bernard S. Black, Agents Watching Agents: The Promise of Institutional Investor Voice, *39 UCLA L. Rev. 811, 815-19 (1992)* (arguing that institutional investors, if active in corporate governance, can improve corporate performance); John C. Coffee, Jr., Liquidity Versus Control: The Institutional Investor as Corporate Monitor, *91 Colum. L. Rev. 1277, 1281-89 (1991)* (expressing the same sentiment, but with reservations); Edward B. Rock, The Logic and (Uncertain) Significance of Institutional Shareholder Activism, *79 Geo. L.J. 445, 469-76 (1990)* (same). For an assessment of the debate between strong and weak proponents of relational investing, see John C. Coffee, The SEC and the Institutional Investor: A Half-Time Report, *15 Cardozo L. Rev. 837 (1994).*

n311. Among others, Professor Kraakman has taken this separation seriously in evaluating the motives and effects of takeovers, and believes that the intuition is "deeply rooted in corporate law and business practice." Kraakman, supra note 9, at 892. In that connection, Professor Kraakman's careful and lucid analysis led him to offer some very wise advice concerning the difficulties for policymakers of bringing financial economics to bear on legal issues that depend on "broad narratives about market behavior." Id. at 941. That advice was "a plea to proceed slowly with legal innovations based on a single account" of market phenomena. Id. I join this plea and particularize it in two ways: policymakers should be cautious in relying on the ECMH and noise theory to evaluate legal rules, and they should be cautious about relying on nonlinear dynamics and chaos theory in doing so as well. Along the same cautionary lines, see Gordon & Kornhauser, supra note 8, at 833 ("In our world, which may be only "close' to the best of all possible worlds, the insights provided by theories of financial markets require patient cultivation before legal policy flowers."), and Langevoort, supra note 5, at 872. Professor Langevoort writes:

While some efficiency properties characterize the securities markets, we simply do not yet know for sure much more than that. This alone should give us pause, for if the sources noted earlier are to be believed, the efficient market theory has become something of an article of doctrinal faith in corporate and securities law. If so, its routine and confident incantations are an embarrassing contrast to the open-mindedness within the economics profession.

Id.